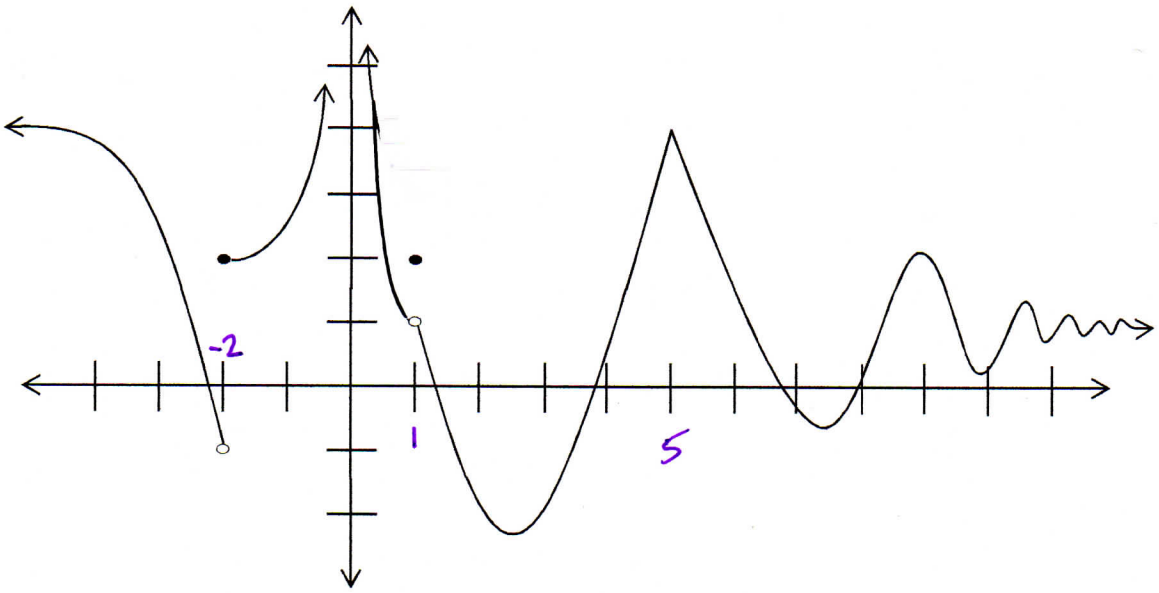
Key

Homework Set 14

Calculating Limits using Graphs

(sections 1.3 & 1.6)

1. For the function $f(x)$, whose graph is shown below, find each value of the quantities listed below the graph. If the quantity does not exist, explain why.



$\lim_{x\to\infty} f(x) =$ 1

$\lim_{x\to -2^-} f(x) =$ −1

$\lim_{x\to -2^+} f(x) =$ 2

$\lim_{x\to -2} f(x) =$ DNE

b/c $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

$f(-2) =$ 2

$\lim_{x\to 0} f(x) =$ ∞

$\lim_{x\to 1^-} f(x) =$ 1

$\lim_{x\to 1^+} f(x) =$ 1

$\lim_{x\to 1} f(x) =$ 1

$f(1) =$ 2

$\lim_{x\to 5} f(x) =$ 4

$\lim_{x\to -\infty} f(x) =$ 4

Where is the function discontinuous?

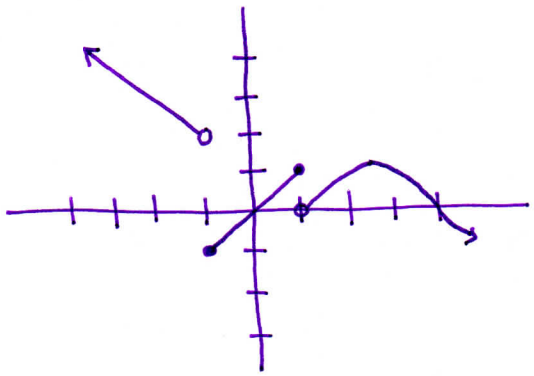
at $x = -2, 0, 1$

On which intervals is the function continuous?

$(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$

2. Sketch the graph of f and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1-x & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ \sin(x-1) & \text{if } x > 1 \end{cases}$$

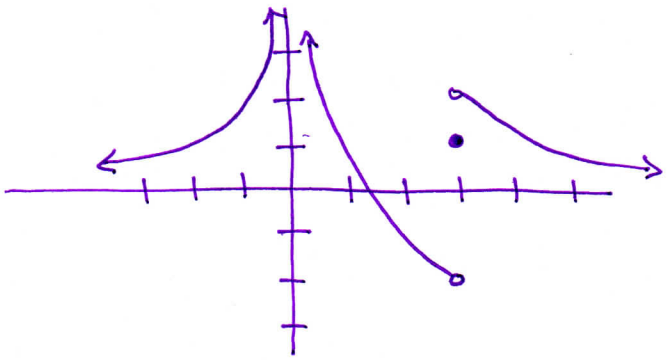


$\lim_{x\to a} f(x)$ exists

for all $a \neq \pm 1$

3. Sketch the graph of a function f that satisfies the following conditions:

$$\lim_{x\to 0} f(x) = \infty, \quad \lim_{x\to 3^-} f(x) = -2, \quad \lim_{x\to 3^+} f(x) = 2,$$
$$f(0) \text{ is undefined, and } f(3) = 1$$



4. Explain in your own words what $\lim_{x\to 2} f(x) = 5$ means. Is it possible for $f(2) = 5$?

This means that as we pick (or look at) x-values which get closer & closer to $x=2$, the y-values $f(x)$ get closer and closer to $y=5$.

Yes, $f(2)=5$ will be true if f is continuous.

5. Explain in your own words what it means for $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 7$. Is it possible for $\lim_{x\to 1} f(x)$ to exist? Why or why not? If it exists, what does $\lim_{x\to 1} f(x)$ equal?

$\lim_{x\to 1^-} f(x) = 3$ means as $x \to 1$ for $x < 1$, $y \to 3$.

$\lim_{x\to 1^+} f(x) = 7$ means as $x \to 1$ for $x > 1$, $y \to 7$.

No, $\lim_{x\to 1} f(x)$ can't exist b/c $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$.