Homework Set 18

The Tangent Line

(sections 2.2 – 2.6, 3.3, & 3.5)

1. The graph of the function $f(x)$ is given below. Use it to find the following derivatives of f.

$f'(-2) = 2$

$f'(-1) = $ DNE (jump)

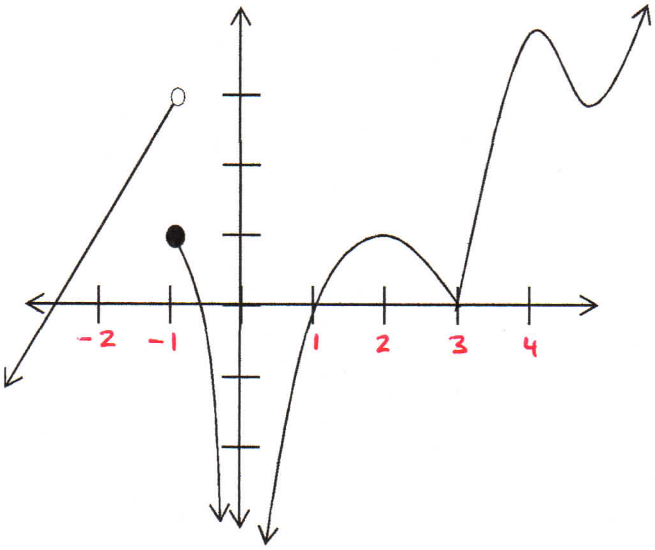
$f'(0) = $ DNE (asymptote)

$f'(1) = 3$

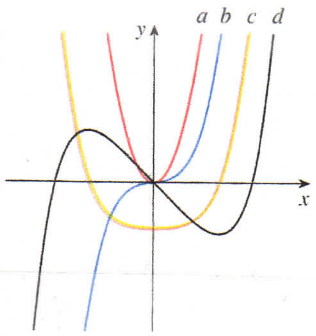
$f'(2) = 0$

$f'(3) = $ DNE (cusp)

$f'(4) = 0$



2. The figure below shows the graphs of the functions f, f', and f''. Identify each curve and explain your choices.



$$\left.\begin{array}{l} d' = c \\ b' = a \\ c' = b \end{array}\right\} \Longrightarrow \begin{array}{l} d = f \\ c = f' \\ b = f'' \\ a = f''' \end{array}$$

For questions 3 – 6, find the equation of the line tangent to the function or equation at the given point.

3. $f(x) = x - \sqrt{x}$ at $(1,0)$

$f'(x) = 1 - \frac{1}{2\sqrt{x}}$

$m = f'(1) = 1 - \frac{1}{2} = 1/2$

$y - 0 = 1/2(x-1)$

$y = 1/2\, x - 1/2$

4. $f(x) = \frac{e^x}{x}$ at $(1, e)$

$$f'(x) = \frac{xe^x - e^x}{x^2} = \frac{(x-1)e^x}{x^2}$$

$$m = f'(1) = \frac{0 \cdot e}{1} = 0$$

$$y - e = 0(x-1)$$

$$y = e$$

5. $y = x\ln(\arctan x)$ at $\left(\frac{\pi}{4}, 0\right)$

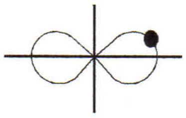
$$\frac{dy}{dx} = \ln(\arctan x) + x \cdot \frac{1}{\arctan x} \cdot \frac{1}{1+x^2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=\pi/4} = \ln(1) + \frac{\pi}{4} \cdot 1 \cdot \frac{1}{1+(\pi/4)^2} = \frac{\pi/4}{1+\pi^2/16} = \frac{4\pi}{16+\pi^2}$$

$$y - 0 = \left(\frac{4\pi}{16+\pi^2}\right)(x - \pi/4)$$

$$y = \frac{4\pi}{16+\pi^2}x - \frac{\pi^2}{16+\pi^2}$$

6. A lemniscate: $2(x^2+y^2)^2 = 25(x^2-y^2)$ at $(3,1)$



$$4(x^2+y^2)(2x+2yy') = 25(2x-2yy')$$

$$4(9+1)(6+2y') = 25(6-2y')$$

$$240 + 80y' = 150 - 50y'$$

$$130y' = -90$$

$$y' = -9/13 \leftarrow m$$

$$y - 1 = -9/13(x-3)$$

$$y = -9/13\,x + 40/13$$

7. For what values of x does the curve $y = 2x^3 + 3x^2 - 12x + 1$ have a horizontal tangent?

$$\frac{dy}{dx} = 6x^2 + 6x - 12 = 0$$

$$6(x^2 + x - 2) = 0$$

$$6(x-1)(x+2) = 0$$

$x = 1$ or $x = -2$

8. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with a slope of 4.

$$\frac{dy}{dx} = 18x^2 + 5 = 4$$

$$18x^2 = -1$$

$$x^2 = -1/18$$

but $x^2 \geq 0$ for any x

So, we can't have $\frac{dy}{dx} = 4$

9. The curve $y = \frac{x}{1+x^2}$ is called a serpentine.

a. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.

$$y' = \frac{(1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$m = \frac{1-9}{(1+9)^2} = \frac{-8}{100} = -\frac{2}{25}$$

$$y - 0.3 = -.08(x-3)$$

$$y = -.08x + .06$$

b. At which points does this curve have a horizontal tangent?

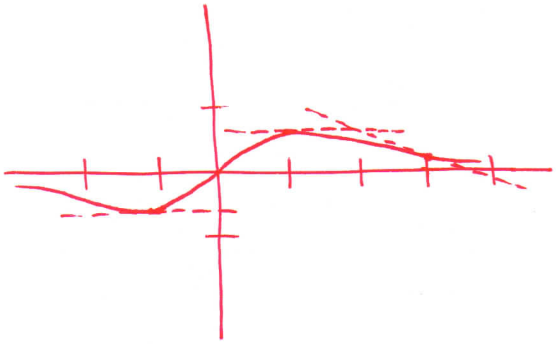
$$\frac{1-x^2}{(1+x^2)^2} = 0$$

$$\Rightarrow 1 - x^2 = 0$$

$$x = \pm 1$$

So $(1, \tfrac{1}{2})$ and $(-1, -\tfrac{1}{2})$

c. Illustrate parts (a) by graphing the curve and its tangent line.



10. The curve $y^2 = x^3 + 3x^2$ is called the Tschirnhausen cubic.

a. Find an equation of the tangent line to this curve at the point $(1, -2)$.

$$2yy' = 3x^2 + 6x$$

$$-4m = 9$$

$$m = -9/4$$

$$y - (-2) = -9/4(x-1)$$

$$y = -9/4\,x + 1/4$$

b. At which points does this curve have a horizontal tangent?

$$y' = 0 \implies 2y \cdot 0 = 3x^2 + 6x$$

$$0 = 3x(x+2)$$

$$x = 0, -2$$

So, $\cancel{(0,0)}$, $(-2, 2)$, $(-2, -2)$

c. Illustrate parts (a) and (b) by graphing the curve and its tangents.