hw set 19
sect 2.8

Find the Linearization line $L(x)$ at the point $x=a$.

1. $f(x) = x^4 + 3x^2$, $a=-1$

point : $(-1, 4)$

$f'(x) = 4x^3 + 6x$

$m = f'(-1) = -4-6 = -10$

$y-(4) = -10(x+1)$

$y = -10x - 10 + 4$

so, $L(x) = -10x - 6$

2. $f(x) = \sin x$, $a = \pi/6$

point : $(\pi/6, 1/2)$

$f'(x) = \cos x$

$m = f'(\pi/6) = \cos(\pi/6) = \sqrt{3}/2$

$y - 1/2 = \sqrt{3}/2\,(x - \pi/6)$

$y = \sqrt{3}/2\, x - \frac{\pi\sqrt{3}}{12} + 1/2$

$L(x) = \frac{\sqrt{3}}{2}x + \frac{6-\pi\sqrt{3}}{12}$

6. Linearize $g(x) = \sqrt[3]{1+x}$ at $a=0$.
Approximate $\sqrt[3]{.95}$ and $\sqrt[3]{1.1}$.
Illustrate by graphing $g(x)$ & $L(x)$.

point : $(0,1)$

$g'(x) = \frac{1}{3}(1+x)^{-2/3}(1) = \frac{1}{3(\sqrt[3]{1+x})^2}$

$m = g'(0) = \frac{1}{3(1)} = 1/3$

$L(x) = \frac{1}{3}x + 1$

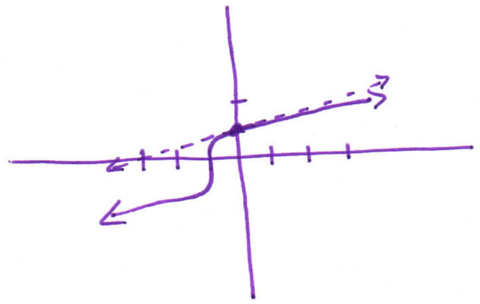
So,

$\sqrt[3]{0.95} = \sqrt[3]{1-.05} = f(-.05)$

$\approx L(-.05) = .98\overline{3}$

$\sqrt[3]{1.1} = \sqrt[3]{1+.1} = f(.1)$

$\approx L(.1) = 1.0\overline{3}$



11. estimate $(1.999)^4$

$f(x) = x^4$ and $a=2$

point : $(2, 16)$

$f'(x) = 4x^3$

$m = f'(2) = 4(2)^3 = 32$

$y-16 = 32(x-2)$

So, $L(x) = 32x - 48$

Then

$(1.999)^4 = f(1.999) \approx L(1.999) = 15.968$

12. estimate $\sqrt[3]{1001}$

$f(x) = \sqrt[3]{x}$ and $a = 1000$

point : $(1000, 10)$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2}$

$m = f'(1000) = \frac{1}{3\cdot 10^2} = \frac{1}{300}$

$y - 10 = \frac{1}{300}(x - 1000)$

So, $L(x) = \frac{1}{300}x + 20/3$

Then

$\sqrt[3]{1001} = f(1001) \approx L(1001) = 10.00\overline{3}$