hw set 20
sect 4.1

1. Difference b/t an abs. min. and a rel. min.

abs min: the lowest point of the function across the entire domain

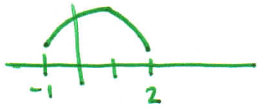
rel min: the lowest point of the function in 1 neighborhood (or interval) of the domain.

For 5-6, find the abs/rel max/min

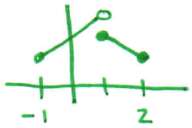
5. rel max: $x = 4, 6$
   abs max: $x = 4$
   rel min: $x = 0, 2, 5$
   abs min: none

6. rel max: $x = 3, 6$
   abs max: none
   rel min: $x = 4, 7$
   abs min: $x = 4$

12. a) Sketch graph: function on $[-1, 2]$ abs max & no other local max



b) sketch graph: function on $[-1, 2]$ Local max & no abs max

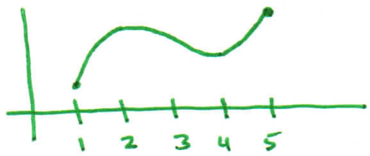


For questions 3-4, identify if the point is an abs max/min, rel max/min, or neither.

3. rel max: $x = a, c, s$
   abs max: $x = s$
   rel min: $x = b, r$
   abs min: $x = r$
   neither: $x = d$

4. rel max: $x = c, r$
   abs max: $x = r$
   rel min: $x = a, d, s$
   abs min: $x = a$
   neither: $x = b$

8. Sketch f where f is continous on $[1, 5]$
   abs min at $x = 1$ Local min at $x = 4$
   abs max at $x = 5$ Local max at $x = 2$



For 23-36, find the critical numbers of the function

24. $f(x) = x^3 + 6x^2 - 15x$

$f'(x) = 3x^2 + 12x - 15$

$0 = 3x^2 + 12x - 15$

$0 = 3(x^2 + 4x - 5) = 3(x-1)(x+5)$

So, $x = 1, -5$

32. $g(x) = x^{1/3} - x^{-2/3}$

$g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$

$0 = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$ multiply by $3x^{5/3}$

$0 = x + 2 \Rightarrow \boxed{x = -2}$

and $g'(x) = \text{dne}$ when $\boxed{x = 0}$

For 37-50, find the abs min/max

38. $f(x) = 5 + 54x - 2x^3$, $[0, 4]$

$f'(x) = 54 - 6x^2$

$0 = -6(x^2 - 9)$

$0 = -6(x+3)(x-3)$

check #s: $x = 0, 3, 4$

$x = 0$: $f(0) = 5$, abs min

$x = 3$: $f(3) = 113$, abs max

$x = 4$: $f(4) = 93$

45. $f(t) = 2\cos t + \sin 2t$, $[0, \pi/2]$

$f'(t) = -2\sin t + 2\cos 2t$

$0 = -2\sin t + 2\cos 2t$

$\sin t = \cos 2t$

So, $t = \pi/6$

$t = 0$: $f(0) = 2$

$t = \pi/6$: $f(\pi/6) = \frac{3\sqrt{3}}{2}$, abs max

$t = \pi/2$: $f(\pi/2) = 0$, abs min

47. $f(x) = xe^{-x^2/8}$, $[-1, 4]$

$f'(x) = e^{-x^2/8} + xe^{-x^2/8}(-\frac{x}{4})$

$0 = e^{-x^2/8}[1 - \frac{1}{4}x^2]$

$1 - \frac{1}{4}x^2 = 0$ $\times(-4)$

$x^2 - 4 = 0$

$x = \pm 2$

$x = -1$: $f(-1) = -e^{-1/8} = -.8825$, abs min

$x = 2$: $f(2) = 2e^{-1/2} = 1.2131$, abs max

$x = 4$: $f(4) = 4e^{-2} = .5413$

49. $f(x) = \ln(x^2+x+1)$, $[-1, 1]$

$f'(x) = \frac{2x+1}{x^2+x+1}$

$0 = \frac{2x+1}{x^2+x+1} \Rightarrow 0 = 2x+1$

So, $x = -1/2$

$x = -1$: $f(-1) = \ln(1) = 0$

$x = -1/2$: $f(-1/2) = \ln(\frac{3}{4}) = -.2877$

$x = 1$: $f(1) = \ln(3) = 1.0986$

abs min at $x = -1/2$

abs max at $x = 1$

50. $f(x) = x - 2\tan^{-1}x$, $[0, 4]$

$f'(x) = x - \frac{2}{1+x^2}$

$0 = x - \frac{2}{1+x^2}$

doesn't factor

$0 = x(1+x^2) - 2$

$0 = x^3 + x - 2 = (x-1)(x^2+x+2)$

So, $x = 1$

$x = 0$: $f(0) = 0$

$x = 1$: $f(1) = -.570796$, abs min

$x = 4$: $f(4) = 1.34836$, abs max

60. $v(t) = 0.001302t^3 - 0.09029t^2 + 23.61t - 3.083$ on $[0, 126]$

estimate the abs max/min values of the acceleration on $[0, 126]$

$a(t) = 0.003906t^2 - 0.18058t + 23.61$

$a'(t) = 0.007812t - 0.18058$

$0 = 0.007812t - 0.18058$

So $t = \frac{0.18058}{0.007812} = 23.11571941$

$t = 0$: $a(0) = 23.61$

$t = 23.1$: $a(23.1157) = 21.52288169$

$t = 126$: $a(126) = 62.868576$

abs max value : 62.8686 at $t = 126$ sec

abs min value : 21.5229 at $t = 23.1157$ sec