hw set 21
sect 4.3

14. $f(x) = x^4(x-1)^3$

a) find critical #s

$f(x) = x^7 - 3x^6 + 3x^5 - x^4$

$f'(x) = 7x^6 - 18x^5 + 15x^4 - 4x^3$

$= x^3(7x^3 - 18x^2 + 15x - 4)$

$= x^3(7x-4)(x-1)^2$

$0 = x^3(7x-4)(x-1)^2$

$\Rightarrow \quad x = 0, 4/7, 1$

b) 2^{nd} Derivative Test

$f''(x) = 42x^5 - 90x^4 + 60x^3 - 12x^2$

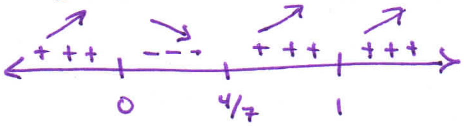
$= 6x^2(x-1)(7x^2 - 8x + 2)$

$f''(0) = 0 \Rightarrow x = 0$ test fails

$f''(1) = 0 \Rightarrow x = 1$ test fails

$f''(4/7) = .2399 > 0 \Rightarrow x = 4/7$ min

c) 1^{st} Derivative Test



max : $x = 0$

min : $x = 4/7$

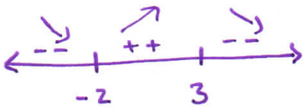
26. $f(x) = 36x + 3x^2 - 2x^3$

a) incr/decr

$f'(x) = 36 + 6x - 6x^2$

$= -6(x^2 - x - 6) = -6(x-3)(x+2)$

$\Rightarrow x = -2, 3$



incr : $(-2, 3)$

decr : $(-\infty, -2) \cup (3, \infty)$

b) max/min values

max : 81 at $x = 3$

min : -2 at $x = -44$

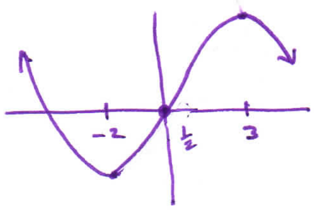
c) cu/cd & inf pts

$f''(x) = 6 - 12x \Rightarrow x = \frac{1}{2}$ inflection pt

cu : $(-\infty, 1/2)$

cd : $(1/2, \infty)$

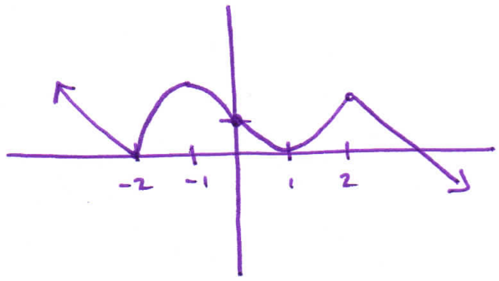
d) sketch



20. $f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$

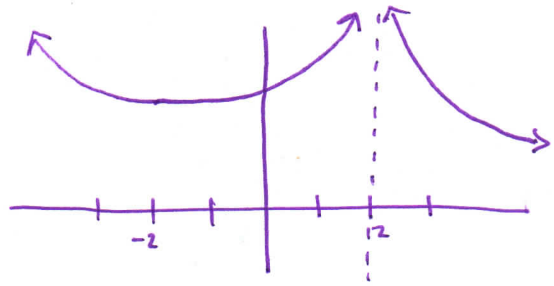
$f'(x) > 0$ if $1 < |x| < 2$, $f'(x) = -1$ if $|x| > 2$

$f''(x) < 0$ if $-2 < x < 0$, inflection pts $(0,1)$



21. $f'(x) > 0$ if $|x| < 2$, $f'(x) < 0$ if $|x| > 2$

$f'(-2) = 0$, $\lim_{x \to 2} |f'(x)| = \infty$, $f''(x) > 0$ if $x \neq 2$



23. see graph

a) incr : $(0,2) \cup (4,6) \cup (8,9)$

decr : $(2,4) \cup (6,8)$

b) max : $x = 2, 6$

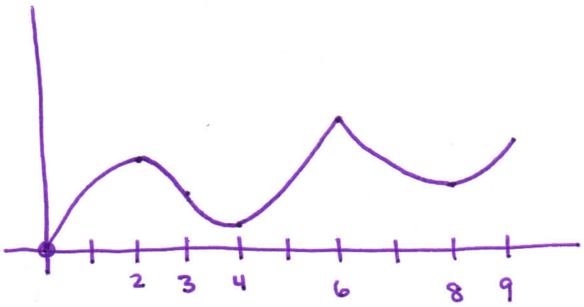
min : $x = 4, 8$

c) cu : $(3,6) \cup (6,9)$

cd : $(0,3)$

d) inf pt(s): $x = 3$

e) sketch f if $f(0) = 0$



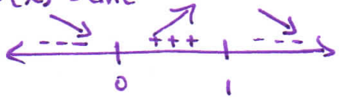
32. $G(x) = 5x^{2/3} - 2x^{5/3}$

a) incr/decr

$$G'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$$
$$= \frac{10}{3}x^{-1/3}(1-x)$$
$$= \frac{10(1-x)}{3\sqrt[3]{x}}$$

$G'(x) = 0 \Rightarrow x = 1$

$G'(x) = \text{dne} \Rightarrow x = 0$



incr: $(0,1)$

decr: $(-\infty,0)\cup(1,\infty)$

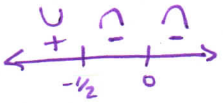
b) max/min

max: 3 at $x=1$

min: 0 at $x=0$

c) Concavity & inflection pts

$$G''(x) = -\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3}$$



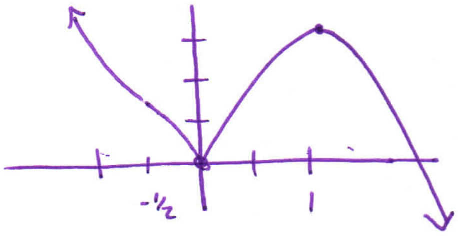
$$= -\frac{10}{9}x^{-4/3}(1+2x)$$
$$= -\frac{10(1+2x)}{9(\sqrt[3]{x^4})}$$

$G''(x) = 0 \Rightarrow x = -\frac{1}{2}$, cu: $(-\infty, -\frac{1}{2})$

$G''(x) = \text{dne} \Rightarrow x = 0$, cD: $(-\frac{1}{2},0)\cup(0,\infty)$

inf pt: $x = -\frac{1}{2}$

d) sketch



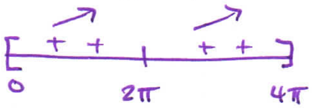
36. $S(x) = x - \sin x$, $0 \le x \le 4\pi$

a) incr/decr

$S'(x) = 1 - \cos x$

$0 = 1 - \cos x$

$\cos x = 1 \Rightarrow x = 0, 2\pi, 4\pi$



incr: $(0,2\pi)\cup(2\pi,4\pi)$

decr: none

b) max/min

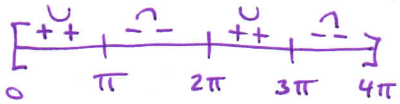
max: 4π at $x = 4\pi$

min: 0 at $x=0$

c) Concavity & Inflection pts

$S''(x) = \sin x$

$0 = \sin x \Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi$

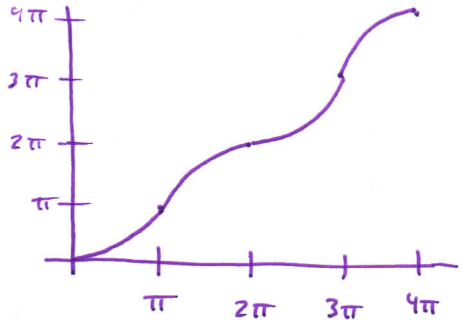


cu: $(0,\pi)\cup(2\pi,3\pi)$

cD: $(\pi,2\pi)\cup(3\pi,4\pi)$

inf pt: $x = \pi, 2\pi, 3\pi$

d) sketch



40. $f(x) = \dfrac{e^x}{1-e^x}$

a) vert/horiz asympt.

horizontal:

$$\lim_{x\to\infty} \frac{e^x}{1-e^x} = \lim_{x\to\infty} \frac{e^x}{-e^x} = -1$$

$$\lim_{x\to-\infty} \frac{e^x}{1-e^x} = \frac{0}{1-0} = 0$$

Vertical:

$$1 - e^x = 0$$

$$e^x = 1$$

VA: $x = 0$

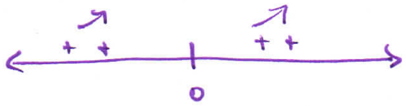
b) incr/decr

$$f'(x) = \frac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^{2x}}{(1-e^x)^2} = \frac{e^x}{(1-e^x)^2}$$

$f'(x) = 0 \Rightarrow e^x = 0 \Rightarrow$ no such x

$f'(x) = \text{dne} \Rightarrow 1 - e^x = 0 \Rightarrow x = 0$



incr: $(-\infty, 0) \cup (0, \infty)$

decr: n/a

c) max/min

none

d) concavity & infl. pts

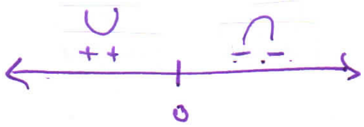
$$f''(x) = \frac{e^x(1-e^x)^2 - e^x \cdot 2(1-e^x)(-e^x)}{(1-e^x)^4}$$

$$= \frac{e^x(1-e^x)[1-e^x+2e^x]}{(1-e^x)^4}$$

$$= \frac{e^x(1+e^x)}{(1-e^x)^3}$$

$f''(x) = 0 \Rightarrow$ none

$f''(x) = \text{dne} \Rightarrow x = 0$



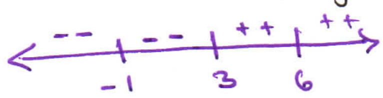
CD: $(0, \infty)$

CU: $(-\infty, 0)$

inf pt: none

45. $f'(x) = (x+1)^2(x-3)^5(x-6)^4$

Where is f increasing?



incr: $(3,6) \cup (6,\infty)$

51. find $f(x) = ax^3 + bx^2 + cx + d$

local max value 3 at $x = -2$

local min value 0 at $x = 1$

$f'(x) = 3ax^2 + 2bx + c$

$f(-2) = 3 \Rightarrow -8a + 4b - 2c + d = 3$

$f'(-2) = 0 \Rightarrow 12a - 4b + c = 0$

$f(1) = 0 \Rightarrow a + b + c + d = 0$

$f'(1) = 0 \Rightarrow 3a + 2b + c = 0$

Solve for a, b, c, d to get

$a = 2/9$

$b = 1/3$

$c = -4/3$

$d = 7/9$

So, $f(x) = \frac{2}{9}x^3 + \frac{1}{3}x^2 - \frac{4}{3}x + 7/9$