7. $y = \frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x$

1) Domain: $(-\infty, \infty)$

2) y-int : $(0,0)$

3) odd function.

4) no asymptotes

5) increasing/decreasing?

$$\frac{dy}{dx} = x^4 - 8x^2 + 16$$
$$0 = (x^2 - 4)^2$$
$$x = \pm 2$$

incr: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
   OR $x \neq \pm 2$

decr: n/a
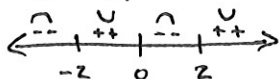
6) max/min?
   none

7) CU/CD/inf pt?
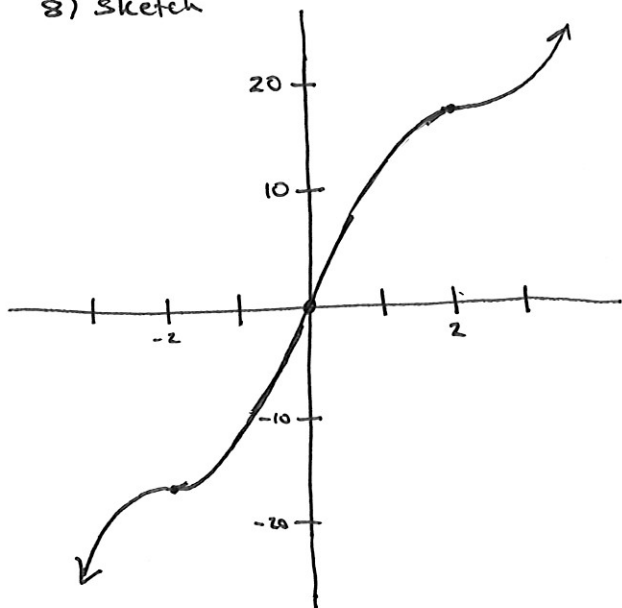
$$\frac{d^2y}{dx^2} = 4x^3 - 16x$$
$$0 = 4x(x^2 - 4)$$
$$x = 0, \pm 2$$

cu: $(-2, 0) \cup (2, \infty)$
cD: $(-\infty, -2) \cup (0, 2)$

8) sketch

12. $y = \frac{x}{x^2 - 9}$
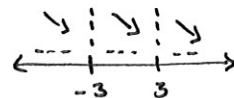
1) Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

2) y-int : $(0, 0)$

3) odd function

4) asymptotes: VA at $x = \pm 3$
               HA at $y = 0$

5) incre/decr ?

$$\frac{dy}{dx} = \frac{1 \cdot (x^2 - 9) - x(2x)}{(x^2 - 9)^2} = \frac{-x^2 - 9}{(x^2 - 9)^2}$$

$$\frac{dy}{dx} = 0 \implies \text{no x's}$$

$$\frac{dy}{dx} = \text{dne} \implies x = \pm 3$$

decr: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

incr: n/a
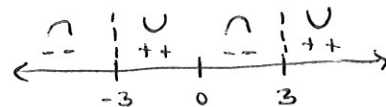
6) max/min?   none

7) CU/CD/inf pt?

$$\frac{d^2y}{dx^2} = \frac{(-2x)(x^2-9)^2 - (-x^2-9) \cdot 2(x^2-9)(2x)}{(x^2-9)^4}$$
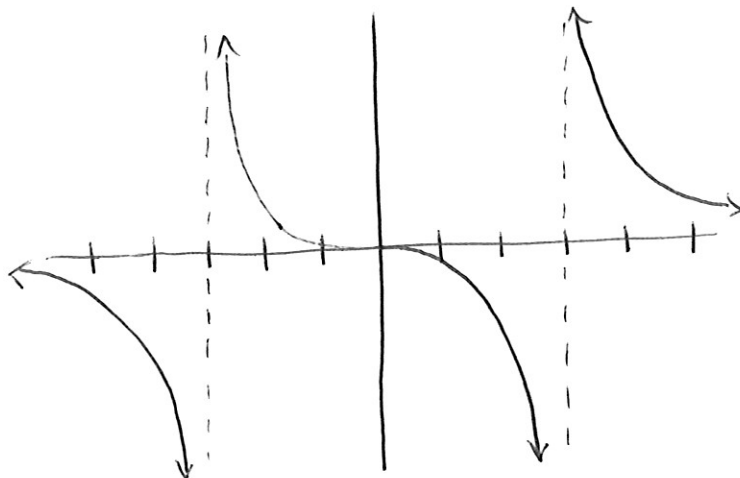
$$= \frac{2x(x^2 + 27)}{(x^2 - 9)^2}$$

$$\frac{d^2y}{dx^2} = 0 \implies x = 0 \quad \& \quad \frac{d^2y}{dx^2} = \text{dne} \implies x = \pm 3$$

cu: $(-3, 0) \cup (3, \infty)$
CD: $(-\infty, -3) \cup (0, 3)$

8) Sketch

30. $y = 2x - \tan x$, $\ -\pi/2 < x < \pi/2$

1) domain: $-\pi/2 < x < \pi/2$
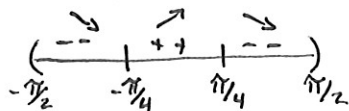
2) y-int: $(0,0)$

3) odd function

4) no asymptotes on domain but at $x = \pm\frac{\pi}{2}$

5) incr/decr?

$$\frac{dy}{dx} = 2 - \sec^2 x$$
$$0 = 2 - \sec^2 x$$
$$\Rightarrow x = \pm \pi/4$$

incr: $(-\pi/4, \pi/4)$

decr: $(-\pi/2, -\pi/4) \cup (\pi/4, \pi/2)$

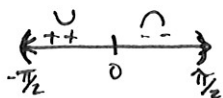6) max: $x = \pi/4 \quad (y = \pi/2 - 1 = .5708)$
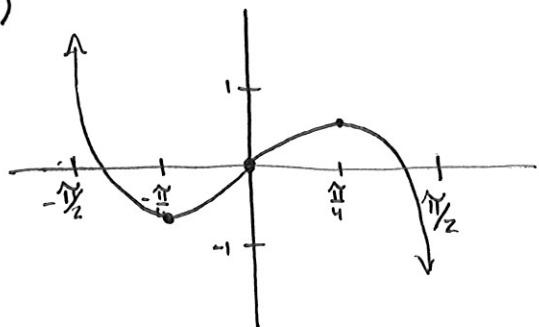min: $x = -\pi/4 \quad (y = -\pi/2 + 1 = -.5708)$

7) cu/cd/inf pts?

$$\frac{d^2 y}{dx^2} = -2\sec x \cdot \sec x \tan x = -2\sec^2 x \tan x$$

$$0 = -2\sec^2 x \tan x$$
$$\Rightarrow x = 0$$

8)

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40. $y = \ln(x^2 - 3x + 2)$

1) domain:
$x^2 - 3x + 2 > 0$
$(x-2)(x-1) > 0$

So domain is $x > 2$ or $x < 1$

2) y-int: $(0, \ln 2)$
x-int: $\left(\frac{3+\sqrt{5}}{2}, 0\right)$ & $\left(\frac{+3-\sqrt{5}}{2}, 0\right)$

3) none

4) When $x \to 1^-$ and $x \to 2^+$

$$\lim_{x \to \infty} \ln(x^2 - 3x + 2) = \infty$$
$$\lim_{x \to -\infty} \ln(x^2 - 3x + 2) = \infty$$

5) incr/decr?

$$\frac{dy}{dx} = \frac{2x-3}{x^2-3x+2} \Rightarrow \frac{dy}{dx} = 0 \text{ if } x = \frac{3}{2}$$

incr: $(2, \infty)$
decr: $(-\infty, 1)$

6) no max/min

7) cu/cd/inf pt

$$\frac{d^2 y}{dx^2} = -\frac{2x^2 - 6x + 5}{(x^2 - 3x + 2)^2}$$

$$\frac{d^2 y}{dx^2} = 0 \Rightarrow \text{never b/c } 2x^2 - 6x + 5 > 0$$

So $\frac{d^2 y}{dx^2} < 0$ for any x

cu: n/a
cd: $(-\infty, 1) \cup (2, \infty)$
inf pt: none

8) sketch

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43. $y = xe^{-1/x}$

1) domain: $x \neq 0$

2) no intercepts

3) none

4) at $x = 0$
$$\lim_{x \to 0^+} xe^{-1/x} = 0 \qquad \lim_{x \to 0^-} xe^{-1/x} = -\infty$$
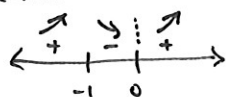$$\lim_{x \to \infty} xe^{-1/x} = \infty \qquad \lim_{x \to -\infty} xe^{-1/x} = -\infty$$

5) incr/decr?
$$y' = e^{-1/x}\left(1 + 1/x\right)$$
$y' = $ dne $\Rightarrow x = 0$
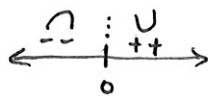$y' = 0 \Rightarrow x = -1$

6) max at $x = -1$

7) cu/cd/inf pt
$$y'' = e^{-1/x} \cdot \left(\frac{1}{x^3}\right)$$
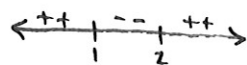$y'' = $ dne $\Rightarrow x = 0$

8) sketch