hw set 23
sect 4.2

For 9–12, verify the function satisfies the MVT and find $x=c$.

9. $f(x) = 2x^2 - 3x + 1$, $[0,2]$

f is continuous on $[0,2]$ ✓
f is differentiable on $(0,2)$ ✓

$f'(x) = 4x - 3$

$\frac{f(2)-f(0)}{2-0} = \frac{3-1}{2} = 1$

$4c - 3 = 1$
then $c = 1$

12. $f(x) = 1/x$, $[1,3]$

f is continuous on $[1,3]$ ✓
f is differentiable on $(1,3)$ ✓

$f'(x) = -1/x^2$

$\frac{f(3)-f(1)}{3-1} = \frac{1/3 - 1}{2} = -1/3$

$-1/c^2 = -1/3$

$c^2 = 3 \Rightarrow c = \sqrt{3}$

For 13–14, find $x=c$. Graph the function, the secant line thru the endpoints, and the tangent line at $x=c$. Are the secant and tangent lines parallel?

13. $f(x) = \sqrt{x}$, $[0,4]$

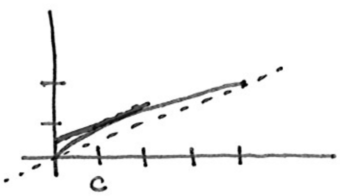
$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{c}} = \frac{\sqrt{4}-\sqrt{0}}{4-0}$

$\frac{1}{2\sqrt{c}} = \frac{2}{4}$

$\frac{1}{2\sqrt{c}} = \frac{1}{2}$

$\sqrt{c} = 1 \Rightarrow c = 1$



yes, they're parallel

14. $f(x) = e^{-x}$, $[0,2]$

$f'(x) = -e^{-x}$

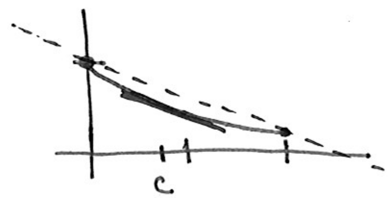
$-e^{-c} = \frac{e^{-2} - e^{-0}}{2-0}$

$-e^{-c} = \frac{1}{2}(e^{-2} - 1)$

$e^{-c} = \frac{1}{2}(1 - e^{-2})$

$-c = \ln\left(\left(\frac{1}{2}\right)(1-e^{-2})\right)$

$c = .83856$



yes, they're parallel

15. Let $f(x) = (x-3)^{-2}$.
Show there is no $x=c$ in $(1,4)$ where $f(4) - f(1) = f'(c)(4-1)$.
Why does this not contradict the MVT?

$f'(x) = -2(x-3)^{-3}$

$\frac{f(4)-f(1)}{4-1} = \frac{1 - 1/4}{3} = \frac{1}{4}$

$-2(x-3)^{-3} = 1/4$

$(x-3)^{-3} = -1/8$

$(x-3)^3 = -8$

$x - 3 = -2$

$x = 1$ so no $1 < c < 4$

It's okay b/c f is not continuous nor differentiable at $x=3$, which is in $(1,4)$

16. Let $f(x) = 2 - |2x - 1|$.
Show there is no $x=c$ such that $f(3) - f(0) = f'(c)(3-0)$.
Why does this not contradict the MVT?

f is continuous on $[0,3]$, but
f is not differentiable on $(0,3)$ b/c $f'(\frac{1}{2})$ = dne
So, the MVT doesn't apply.

if $x > 1/2$, $f'(x) = -2$
if $x < 1/2$, $f'(x) = 2$

$\frac{f(3)-f(0)}{3-0} = \frac{-3-1}{3} = -4/3 = -1.\overline{3}$

So, no such $x=c$ where
$f'(c) = -4/3$

23. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$,
how small can $f(4)$ possibly be?

Use MVT:

$$f(b) - f(a) = (b-a) f'(c)$$
$$f(4) - f(1) = (4-1) f'(c)$$
$$f(4) - 10 = 3f'(c)$$
$$f(4) = 10 + 3f'(c)$$

So, $f(4) \geq 10 + 3(2) = 16$

smallest is $f(4) = 16$

24. Suppose $3 \leq f'(x) \leq 5$ for all x.
Show $18 \leq f(8) - f(2) \leq 30$

consider the interval $[2, 8]$

$$3 \leq f'(x) \leq 5$$
$$\downarrow$$
$$3 \leq f'(c) \leq 5$$
$$3 \leq \frac{f(8) - f(2)}{8-2} \leq 5$$
$$18 \leq f(8) - f(2) \leq 30$$

25. Does there exist a function f such that
$f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x

consider the interval $[0, 2]$

$f'(x) \leq 2$ for all x means: f is continuous on $[0,2]$
& f is differentiable on $(0,2)$

apply the MVT:

$$f'(c) = \frac{f(2) - f(0)}{2-0} \quad \text{for some } 0 < c < 2$$

$$f'(c) = \frac{4-(-1)}{2} = 5/2 = 2.5$$
$$\uparrow$$

not possible b/c $f'(x) \leq 2$ for all x (even $x = c$)

So, no such function f.