hw set 24
sect 4.5

2. 2 numbers with difference of 100 & minimum product

numbers: x, y

relation: $x - y = 100$

Sum $= x \cdot y$

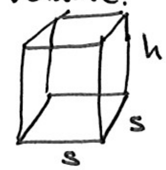
$S(x) = x \cdot (x-100) = x^2 - 100x$

$S'(x) = 2x - 100$

$0 = 2x - 100 \Rightarrow x = 50$

and $y = -50$

11. $1200\ cm^2$ material available to make a box w/ square base & open top, maximize volume.



SA: $1200 = s^2 + 4sh$

Vol $= s^2 h$

$4sh = 1200 - s^2$

$h = \frac{300}{s} - \frac{1}{4}s$

$V(s) = s^2\left(\frac{300}{s} - \frac{1}{4}s\right) = 300s - \frac{1}{4}s^3$

$V'(s) = 300 - \frac{3}{4}s^2 = 0$

$s^2 = 400$

$s = 20 \longrightarrow h = 10$

max vol $= s^2 h = 4000$

15. Find a point on $y = 2x+3$ that is closest to the origin.

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d^2 = (x-0)^2 + ((2x+3)-0)^2$

$= x^2 + (2x+3)^2$

$= x^2 + 4x^2 + 12x + 9$

$= 5x^2 + 12x + 9$

$2d \cdot d' = 10x + 12$

set $d' = 0$:

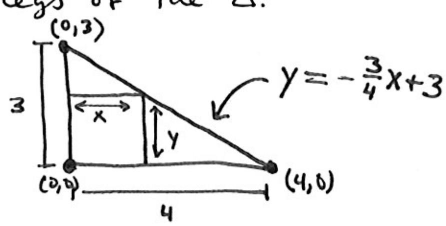
$0 = 10x + 12 \Rightarrow x = -6/5$

then $y = 3/5$

$\therefore (-6/5, 3/5)$

22. Find area of largest rectangle inscribed in a right Δ with legs 3 cm & 4 cm if 2 sides of $\square$ lie along the legs of the Δ.

place on x-y plane



Area $= xy$

$A(x) = x\left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x$

$A'(x) = -\frac{3}{2}x + 3$

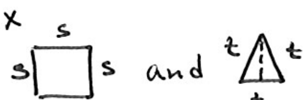
$0 = -\frac{3}{2}x + 3 \Rightarrow x = 2$

then $y = 3/2$

max area: $3\ cm^2$

27. Wire 10cm long – 2 pieces ($\square$ & Δ) How should the wire be cut so the total area is a

a) max



perimeter: $4s + 3t = 10$

Area $= s^2 + \frac{1}{2}t \cdot \left(\sqrt{3} \cdot \frac{t}{2}\right)$

$A(t) = \left(\frac{10-3t}{4}\right)^2 + \frac{\sqrt{3}}{4}t^2$

$= \frac{100 - 60t + 9t^2}{16} + \frac{\sqrt{3}}{4}t^2$

$= \frac{25}{4} - \frac{15}{4}t + \frac{9+4\sqrt{3}}{16}t^2$

$A'(t) = -\frac{15}{4} + \frac{9+4\sqrt{3}}{8}t = 0$

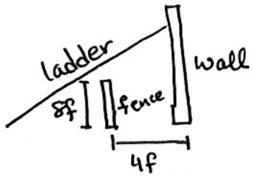
$\Rightarrow t = \frac{30}{9+4\sqrt{3}} = 1.8835$

$3t = \frac{90}{9+4\sqrt{3}} = 5.65035$

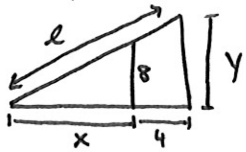
$s = \frac{10 - 3\left(\frac{30}{9+4\sqrt{3}}\right)}{4} = 1.087411$

$4s = 4.3496$

28.



want shortest ladder



$$\frac{8}{x} = \frac{y}{x+4} \Rightarrow y = 8 + \frac{32}{x}$$

$$\ell^2 = y^2 + (x+4)^2$$

$$\ell^2 = \left(8+\frac{32}{x}\right)^2 + (x+4)^2$$

$$2\ell\ell' = 2\left(8+\frac{32}{x}\right)\left(-\frac{32}{x^2}\right) + 2(x+4)$$

Set $\ell' = 0$

$$0 = 2\left(8+\frac{32}{x}\right)\left(-\frac{32}{x^2}\right) + 2x+8$$

$$0 = -512 - \frac{2048}{x} + 2x^3 + 8x^2$$

$$0 = 2x^4 + 8x^3 - 512x - 2048$$

$$0 = x^4 + 4x^3 - 256x - 1024$$

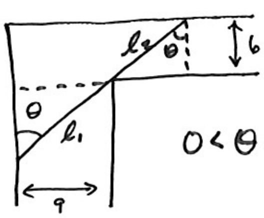
$$0 = x^3(x+4) - 256(x+4)$$

$$0 = (x^3 - 256)(x+4)$$

$$\Rightarrow \cancel{x=-4} \text{ or } x = \sqrt[3]{256} = 6.3496$$

$$\Rightarrow \ell = 16.64775 \text{ ft}$$

54.



$0 < \theta < \pi/2$

$$\sin\theta = \frac{9}{\ell_1} \Rightarrow \ell_1 = \frac{9}{\sin\theta}$$

$$\cos\theta = 6/\ell_2 \Rightarrow \ell_2 = \frac{6}{\cos\theta}$$

$$\ell(\theta) = \frac{9}{\sin\theta} + \frac{6}{\cos\theta}$$

$$\ell'(\theta) = -\frac{9\cos\theta}{\sin^2\theta} + \frac{6\sin\theta}{\cos^2\theta} = 0$$

$$0 = -9\cos^3\theta + 6\sin^3\theta$$

$$\Rightarrow 3\cos^3\theta = 2\sin^3\theta$$

$$\Rightarrow \tan\theta = \sqrt[3]{\frac{3}{2}}$$

$$\Rightarrow \theta = .85277$$

So, $\ell(\theta) = 21.07$

45. Baseball team:

stadium 55,000 max

ticket \$10 → avg att 27,000

ticket \$8 → avg att 33,000

a) find demand function (linear)

$p(x)$ = price when sell x units

(27,000; \$10) & (33,000, \$8)

$$m = \frac{10-8}{27{,}000-33{,}000} = -\frac{1}{3000}$$

$$y - 10 = -\frac{1}{3000}(x - 27000)$$

$$y - 10 = -\frac{1}{3000}x + 9$$

$$\Rightarrow p(x) = -\frac{1}{3000}x + 19$$

b) how should tickets be set to maximize Revenue?

$$R(x) = x\,p(x) = -\frac{1}{3000}x^2 + 19x$$

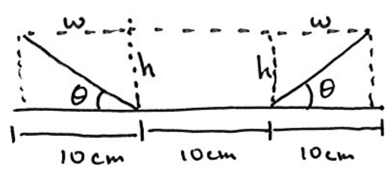
$$R'(x) = -\frac{1}{1500}x + 19 = 0$$

$$x = 19(1500) = 28{,}500$$

ticket price:

$$p(28{,}500) = \$9.50$$

56.



gutter formed by bending the meter sheet in thirds

find θ where gutter can carry the maximum amount of water.

$$\sin\theta = \frac{h}{10} \qquad \cos\theta = \frac{w}{10}$$

Area of cross section:

$$A = 2\left[\tfrac{1}{2}wh\right] + 10h = wh + 10h$$

$$= 10\cos\theta \cdot 10\sin\theta + 10\cdot 10\sin\theta$$

$$= 100\sin\theta(\cos\theta + 1)$$

$$A' = 100\cos\theta(\cos\theta+1) + 100\sin\theta(-\sin\theta)$$

$$= 100\cos\theta + 100\cos^2\theta - 100\sin^2\theta$$

$$= 100\left[\cos\theta + 2\cos^2\theta - 1\right]$$

$$0 = 100(2\cos\theta - 1)(\cos\theta + 1)$$

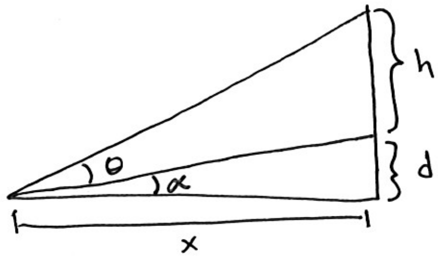
$$\Rightarrow 2\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \frac{\pi}{3}$$

$$\cos\theta + 1 = 0 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$$

<u>but</u> $0 \le \theta \le \pi/2$

$$\therefore \theta = \pi/3$$

58.



h & d are fixed.

find x in order to maximize θ

$$\tan(\theta+\alpha) = \frac{h+d}{x} \quad \text{and} \quad \tan\alpha = \frac{d}{x}$$

$$\theta+\alpha = \arctan\left(\frac{h+d}{x}\right)$$

$$\theta(x) = \arctan\left(\frac{h+d}{x}\right) - \arctan\left(\frac{d}{x}\right)$$

$$\theta'(x) = \frac{\left(-\frac{h+d}{x^2}\right)}{1+\left(\frac{h+d}{x}\right)^2} - \frac{\left(-d/x^2\right)}{1+\left(d/x\right)^2} = -\frac{h+d}{x^2+(h+d)^2} + \frac{d}{x^2+d^2}$$

want $\theta'(x)=0$:

$$0 = -\frac{h+d}{x^2+(h+d)^2} + \frac{d}{x^2+d^2}$$

$$\begin{aligned} 0 &= -(h+d)(x^2+d^2) + d\left[x^2+(h+d)^2\right] \\ &= -(h+d)x^2 - (h+d)d^2 + dx^2 + d(h+d)^2 \\ &= -hx^2 + (h+d)\left[-d^2 + d(h+d)\right] \\ &= -hx^2 + (h+d)(dh) \end{aligned}$$

$$hx^2 = (h+d)dh$$

$$x^2 = d(h+d)$$

$$x = \sqrt{(h+d)d}$$