hw set 25

sect 3.4

4. Bacteria Culture: $\frac{1}{P} \cdot \frac{dP}{dt} = k$

$P(2) = 400$ and $P(6) = 25{,}600$.

a) What is k? (as a %)

$P(t) = P_o e^{kt}$

$t=2$: $400 = P_o e^{2k}$

$t=6$: $25{,}600 = P_o e^{6k}$

$25{,}600 = P_o \left(\frac{400}{P_o}\right)^3$ b/c $e^{6k} = (e^{2k})^3$

$25{,}600 = \frac{64000000}{P_o^2}$

$P_o^2 = \frac{64000000}{25600}$

$P_o = 50$

then $400 = 50e^{2k}$

$8 = e^{2k}$

$\ln 8 = 2k$

$\frac{3}{2}\ln 2 = \frac{1}{2}\ln 8 = k$

So, $K = 1.03972$

or $k = 103.97\%$

b) What is $P(0) = P_o$?

$P_o = 50$

c) Find $P(t)$.

$P(t) = 50e^{(3t\ln 2)/2}$

or $P(t) = 50e^{(1.03972)t}$

d) Find $P(4.5)$.

$P(4.5) = 5381.737058$

So, the number of cells is 5381

e) Find the rate of growth after 4.5 hrs (ie: $P'(4.5)$)

since $\frac{1}{P} \cdot \frac{dP}{dt} = k$, $\frac{dP}{dt} = kP$

$P'(4.5) = k \cdot P(4.5)$

$= (1.03972)(5381.737)$

$= 5595.503802$

f) Find t where $P(t) = 50{,}000$

$50e^{(3t\ln 2)/2} = 50{,}000$

$(3t\ln 2)/2 = \ln(1000)$

$t = \frac{2\ln(1000)}{3\ln 2} = 6.6439$ hrs

5. World Population

	Year	Pop.	Year	Pop.	
t=0	1750	790	1900	1650	t=3
t=1	1800	980	1950	2560	t=4
t=2	1850	1260	2000	6080	t=5

a) Use years 1750 & 1800 & $P(t) = P_o e^{kt}$ to predict pop. in 1900 & 1950. Compare with actual figures.

$P(0) = 790$ & $P(1) = 980$

$\hookrightarrow 790 = P_o$

$980 = 790e^k \Rightarrow k = \ln\left(\frac{98}{79}\right) = .2155$

$P(t) = 790e^{(.2155)t}$

1900: $P(3) = 1508.08$, actual is 1650

1950: $P(4) = 1870.78$, actual is 2560

b) Use years 1850 & 1900 & $P(t) = P_o e^{kt}$ to predict pop. in 1950. Compare with actual figures.

$P(2) = 1260$ & $P(3) = 1650$

$1260 = P_o e^{2k} \longrightarrow e^k = \left(\frac{1260}{P_o}\right)^{1/2}$

$1650 = P_o e^{3k}$

$\hookrightarrow 1650 = P_o \left(\frac{1260}{P_o}\right)^{3/2}$

$1650 = P_o^{-1/2}(1260)^{3/2}$

$P_o = 734.757$

$e^k = 1.3095$

$k = .26966$

$P(t) = (734.757)e^{(.26966)t}$

$t=4$: $P(4) = 2160.714$, actual is 2560

c) Use years 1900 & 1950 & $P(t) = P_o e^{kt}$ to predict pop. in 2000. Compare with actual fig. & explain.

$P(3) = 1650$ & $P(4) = 2560$

$1650 = P_o e^{3k} \longrightarrow e^k = \left(\frac{1650}{P_o}\right)^{1/3}$

$2560 = P_o e^{4k}$

$\hookrightarrow 2560 = P_o \left(\frac{1650}{P_o}\right)^{4/3}$

$2560 = P_o^{-1/3}(1650)^{4/3}$

$P_o = 441.7899996$

$e^k = 1.55\overline{15}$

$K = .43923$

$P(t) = (441.79)e^{(.439)t}$

$t=5$: $P(5) = 3971.\overline{87}$

Life expectancy has increased so the previous models won't necessarily work

8. Strontium-90 has a half-life of 28 days.

a) $M_0 = 50$, find $M(t) = m_0 e^{kt}$

$M(28) = 25$

So $\frac{1}{2} = e^{28K}$

$-\ln 2 = 28k$

$K = -\frac{\ln 2}{28} = -.024755$

$M(t) = 50 e^{(-.024755)t}$

b) find $M(40)$.

$M(40) = 18.5749$

c) find t where $M(t) = 2$

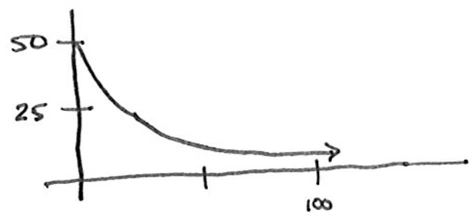
$2 = 50 e^{(-.024755)t}$

$\ln\left(\frac{1}{25}\right) = -\frac{\ln 2}{28} \cdot t$

$\frac{-2\ln 5}{-\ln 2} \cdot 28 = t$

So, $t = 130.0279733$

d) sketch $M(t)$



20. a) find t for $P = 2A$ if $r = .06$ & $P = Ae^{rt}$

$2A = Ae^{(.06)t}$

$2 = e^{(.06)t}$

$t = \frac{\ln 2}{.06} = 11.5525$

b) What is the equivalent annual interest rate?

$P = A\left(1 + \frac{r}{1}\right)^{1 \cdot t}$ find r which gives

$2A = A(1+r)^t$

$2^{1/t} = 1 + r$

$r = 2^{1/t} - 1 = 2^{\frac{1}{11.5525}} - 1$

So, $r = .0618365 \approx 6.18\%$

13. Turkey - oven : $\frac{dT}{dt} = k(T - T_s)$

Then $T(t) = T_s + Ae^{kt}$ where

$T_s = 75°F$ and $T(0) = 185°F$

a) $T(1/2) = 150°F$, find $T(3/4)$

$150 = 75 + Ae^{k(1/2)}$ & $185 = 75 + A$

$75 = Ae^{k/2} \longleftarrow 110 = A$

$75 = 110e^{k/2}$

$k = 2\ln(75/110) = -.7659845$

$T(t) = 75 + 110e^{(-.766)t}$

$T(3/4) = 136.9292$

b) find t when $T(t) = 100$

$100 = 75 + 110e^{kt}$

$\frac{25}{110} = e^{kt}$

$t = \frac{1}{k}\ln(25/110) = 1.9342$

14. Murder : $\frac{dT}{dt} = k(T - T_s)$ so

$T(t) = T_s + Ae^{kt}$ where

$T(1.5) = 32.5$ & $T(2.5) = 30.3$ & $T_s = 20$

find the time (t-adjusted) when $T(t) = 37$.

$t = 1.5$: $32.5 = 20 + Ae^{k(1.5)}$

$t = 2.5$: $30.3 = 20 + Ae^{k(2.5)}$

$\Rightarrow 12.5 = Ae^{3/2k} \longrightarrow \left(\frac{12.5}{A}\right)^{2/3} = e^k$

$10.3 = Ae^{5/2k}$

$\hookrightarrow 10.3 = A\left(\frac{12.5}{A}\right)^{5/3}$

$10.3 = A^{-2/3}(12.5)^{5/3}$

$A = 16.71164477$

$e^k = .824$

$k = -.19358$

want: $37 = 20 + (16.7)e^{(-.19)t}$

$17 = (16.7)e^{(-.19)t}$

$.017 = (-.19)t$

$t = -.08837$

$\Rightarrow$ 11:54:30 am