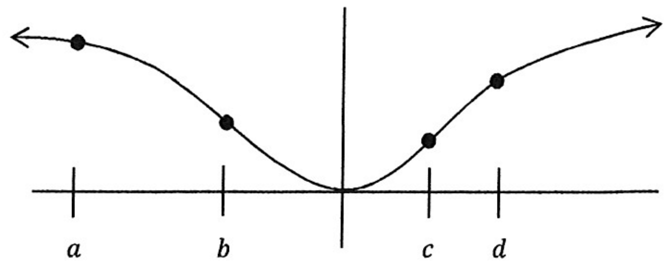
Key

Homework Set 26

Sect 4.6: Newton's Method

1. For which of the initial approximations: $x_1 = a, x_1 = b, x_1 = c$, and $x_1 = d$ will Newton's method work so that the subsequent x_n approximations will approach the root of the equation $f(x) = 0$?



for $x_1 = b, c, d$

if $x_1 = a$, it'll take you farther away from $x = 0$.

2. Use Newton's method to approximate $\sqrt[5]{20}$ correct to 8 decimal places.

$x = \sqrt[5]{20}$

$x^5 = 20$

$x^5 - 20 = 0$

$f(x) = x^5 - 20$

$f'(x) = 5x^4$

$x_1 = 2$ (pick 1 or 2)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.85$

$x_3 = 1.821486137$

$x_4 = 1.820565136$

$x_5 = 1.820564203$

$x_6 = 1.820564203$

So, $\sqrt[5]{20} \approx 1.820564203$

3. Use Newton's method to approximate the root of $x^4 - 2x^3 + 5x^2 - 6 = 0$ in the interval [1,2] correct to 6 decimal places.

$f(x) = x^4 - 2x^3 + 5x^2 - 6$

$f'(x) = 4x^3 - 6x^2 + 10x$

$x_1 = 1$ (pick 1 or 2)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.25$

$x_3 = 1.218214286$

$x_4 = 1.217562422$

$x_5 = 1.217562155$

$x_6 = 1.217562155$

So, the root is

$x = 1.217562$

4. Use Newton's method to approximate the positive root of $3\sin x = x$ correct to 6 decimal places.

$f(x) = 3\sin x - x$

$f'(x) = 3\cos x - 1$

$x_1 = 2$ pick (2 or 3)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.323732061$

$x_3 = 2.279594822$

$x_4 = 2.278862867$

$x_5 = 2.27886266$

$x_6 = 2.27886266$

So, the root is

$x = 2.27886266$

5. For the following questions, use the equation $x^3 - 15x + 5 = 0$ on the interval $[-2,2]$.

a. Use the Intermediate Value Theorem to show that there is at least one root in the interval.

$f(x) = x^3 - 15x + 5$

$f(-2) = 27$ and $f(2) = -17$

Then by the IVT, there is a c in $(-2,2)$ where $f(c) = 0$
(ie: c is ~~a~~ root)

b. Use the Mean Value Theorem to show that there is exactly one root.

$f'(x) = 3x^2 - 15$

way 1:

for $-2 \le x \le 2$,

$f'(x) < 0$ (always decreasing)

$\therefore$ exactly 1 root

way 2: Suppose $x=a$ & $x=b$ are both roots of $x^3 - 15x + 5 = 0$. Then there is a $x = c$ where

$f'(c) = \frac{f(a) - f(b)}{a-b} = \frac{0}{a-b} = 0$

so $3c^2 - 15 = 0 \Rightarrow c = \pm\sqrt{5}$

but $\sqrt{5} > 2$, so only 1 root

c. Use Newton's method to find this root correct to 6 decimal places.

$f(x) = x^3 - 15x + 5$

$f'(x) = 3x^2 - 15$

$x_1 = 0$ (pick 0 or 1)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1/3$

$x_3 = .3358585859$

$x_4 = .3358590219$

$x_5 = .3358590219$

So, the root is

$x = .3358590219$