Homework Set 4

The Chain Rule

(sect 2.5)

Write the composite function in the form: $f(g(x))$. [Clearly identify the inner and outer functions.] Then find the derivative $\frac{dy}{dx}$.

1. $y = e^{x^2-3x}$

$\frac{dy}{dx} = (2x-3\cancel{x})e^{x^2-3x}$

$u = g(x) = x^2-3x$ $\quad u' = 2x-3$

$f(u) = e^u$ $\quad f'(u) = e^u$

2. $y = \sin(\tan\ \ x)$

$\frac{dy}{dx} = \cos(\tan\ x)\cdot\sec^2(\ x)$

$u = g(x) = \tan\ \ x$ $\quad u' = \sec^2\ x$

$f(u) = \sin u$ $\quad f'(u) = \cos u$

For some composite functions, the derivative can be found without using the chain rule. This is only possible if the function can be rewritten in such a way that another rule can be used.

3. $y = (1-2t)^2$

   a. If possible, compute $\frac{dy}{dt}$ without using the chain rule.

$y = (1-2t)^2 = 1-4t+4t^2$

$\frac{dy}{dt} = -4+8t$

   b. Find $\frac{dy}{dt}$ using the chain rule.

$\frac{dy}{dt} = 2(1-2t)^1\cdot(-2) = -4(1-2t)$

   c. Are the answers for part (a) and part (b) the same function?

yes, since $-4(1-2t) = -4+8t$

Compute the derivatives of the following functions. Simplify your answers where applicable (hint: look for compound fractions and trig functions).

4. e^{5x}

$5e^{5x}$

5. $\sin x^2$

$2x\cos(x^2)$

6. $\cos^2\theta$

$2\cos\theta(-\sin\theta)$

$= -2\cos\theta\sin\theta$

OR $-\sin 2\theta$

7. $\ln x^7 + \ln 7x$

$7/x + 1/x$

$= 8/x$

8. $(\ln t)^7$

$7(\ln t)^6 \cdot \frac{1}{t}$

9. $(w^2 - 3w + 1)^{12}$

$12(w^2-3w+1)^{11}(2w-3)$

10. $\ln(\sec y + \tan y)$

$\frac{\sec y\tan y + \sec^2 y}{\sec y + \tan y}$

$= \sec y$

11. $\ln(x^{29} - 3x^{15} + 7x^6 + 3x^2 - 1)$

$\frac{29x^{28} - 45x^{14} + 42x^5 + 6x}{x^{29} - 3x^{15} + 7x^6 + 3x^2 - 1}$

12. $\sqrt{\arctan x}$

$\frac{1}{2\sqrt{\arctan x}} \cdot \frac{1}{x^2+1}$

13. $\arctan\left(\frac{t+1}{3}\right) = 3\cdot\left[\frac{1}{3}\arctan\left(\frac{t+1}{3}\right)\right]$

$\frac{3}{(t+1)^2 + 9}$

14. $(r^2 - (4-r)^5)^3$

$3\left[r^2-(4-r)^5\right]^2\cdot\left[2r+5(4-r)^4\right]$

15. $\left(4 - e^{x^2} + 5\ln x\right)^6$

$6(4-e^{x^2}+5\ln x)^5(-2xe^{x^2} + 5/x)$

16. $\ln(\sec x^4)$

$\frac{4x^3\sec x^4\tan x^4}{\sec x^4} = 4x^3\tan(x^4)$

17. $2^{5^{x^2}} - \log_2(\cos x)$

$2^{5^{x^2}}\ln 2\cdot\frac{d}{dx}\left(5^{x^2}\right) - \frac{1}{\cos x\ln 2}\cdot(-\sin x)$

$= 2^{5^{x^2}}\ln 2\cdot 5^{x^2}\ln 5\cdot 2x + \frac{\tan x}{\ln 2}$

18. $\arcsin(\cos\theta)$

$$\frac{1}{\sqrt{1-\cos^2\theta}} \cdot (-\sin\theta)$$

$$= \frac{1}{\sin\theta}(-\sin\theta)$$

$$= -1$$

$u = \cos\theta$ $u' = -\sin\theta$

$f(u) = \arcsin u$ $f'(u) = \frac{1}{\sqrt{1-x^2}}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 1-\cos^2\theta = \sin^2\theta$

Use the definition of the Chain rule to answer the following questions.

19. If $h(x) = \sqrt{5+2f(x)}$ where $f(0) = 2$ and $f'(0) = -6$, find $h'(0)$.

$$h'(x) = \frac{1}{2\sqrt{5+2f(x)}} \cdot (2f'(x)) = \frac{f'(x)}{\sqrt{5+2f(x)}}$$

$$h'(0) = \frac{f'(0)}{\sqrt{5+2\cdot f(0)}} = \frac{-6}{\sqrt{5+2\cdot 2}} = \frac{-6}{3} = -2$$

20. A table for the values for f, g, f', g' at the given values is given below:

t	$f(t)$	$g(t)$	$f'(t)$	$g'(t)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. Let $h(t) = f(g(t))$. Compute $h'(1)$.

$$h'(1) = f'(g(1)) \cdot g'(1)$$
$$= f'(2) \cdot 6$$
$$= 5 \cdot 6 = 30$$

b. Let $k(t) = f(f(t))$. Compute $k'(2)$.

$$k'(2) = f'(f(2)) \cdot f'(2)$$
$$= f'(1) \cdot f'(2)$$
$$= 4 \cdot 5$$
$$= 20$$