Homework Set 5

The Product & Quotient Rules

(sect 2.4)

1. Compute: $D\big((x^2+1)(2-3x)\big)$
   a. Find the derivative by using the product rule.

      $(2x)(2-3x)+(x^2+1)(-3)$

   b. Find the derivative by multiplying out the function first and not using the product rule.

      $D((x^2+1)(2-3x)) = D(-3x^3+2x^2-3x+2)$

      $= -9x^2+4x-3$

   c. Do you get the same function?

      yes, b/c $2x(2-3x)-3(x^2+1) = 4x-6x^2-3x^2-3 = -9x^2+4x-3$

2. Compute: $\frac{d}{dt}\left(\frac{3t^5-2t\sqrt{t}+t}{t^2}\right) = \frac{d}{dt}\left(\frac{3t^5-2t^{3/2}+t}{t^2}\right)$
   a. Find the derivative by using the quotient rule.

      $$\frac{(15t^4-3t^{1/2}+1)t^2-(3t^5-2t^{3/2}+t)\cdot 2t}{t^4}$$

      $$= \frac{15t^5-3t^{3/2}+t-6t^5+4t^{3/2}-2t}{t^3} = \frac{9t^4+t^{1/2}-t}{t^2}$$

   b. Find the derivative by simplifying the function first and not using the quotient rule.

      $$\frac{d}{dt}\left(\frac{3t^5-2t^{3/2}+t}{t^2}\right) = \frac{d}{dt}\left(3t^3-2t^{-1/2}+t^{-1}\right) = 9t^2+t^{-3/2}-t^{-2}$$

   c. Do you get the same function?

      yes, because $\frac{9t^4+t^{1/2}-t}{t^2} = 9t^2+t^{-3/2}-t^{-2}$

Compute the derivatives of the following functions.

3. $x\sin x$

   $\sin x + x\cos x$

4. $x^4 \ln 5x$

   $4x^3\ln 5x + x^4\cdot\frac{1}{x}$

   $= 4x^3\ln 5x + x^3$ OR $x^3(1+4\ln 5x)$

5. $\sqrt{x}\cdot e^{-x}$

$$\frac{1}{2\sqrt{x}}e^{-x}+\sqrt{x}\left(-e^{-x}\right) = \frac{1}{2e^{x}\sqrt{x}}-\frac{\sqrt{x}}{e^{x}}$$

6. $\frac{x^2}{e^x}$

$$\frac{2x\cdot e^{x}-x^{2}e^{x}}{(e^{x})^{2}} = \frac{2x-x^{2}}{e^{x}}$$

7. $\frac{x}{x^2+1}$

$$\frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$$

8. $\frac{1-5x}{3x+2}$

$$\frac{(-5)(3x+2)-(1-5x)(3)}{(3x+2)^2} = \frac{-13}{(3x+2)^2}$$

9. $\frac{e^w-e^{-w}}{e^w+e^{-w}}$

$$\frac{(e^{w}+e^{-w})^2-(e^{w}-e^{-w})^2}{(e^{w}+e^{-w})^2} = \frac{e^{2w}+2+e^{-2w}-(e^{2w}-2+e^{-2w})}{(e^{w}+e^{-w})^2} = \frac{4}{(e^{w}+e^{-w})^2}$$

10. $x^3 \sin x \cos 3x$

$$3x^2\sin x\cos 3x + x^3\cos x\cos 3x - 3x^3\sin x\sin 3x$$

11. $\left(\frac{xe^{3x}}{x^2-4}\right)^5$

$$5\left(\frac{xe^{3x}}{x^2-4}\right)^4\cdot\frac{[e^{3x}+3xe^{3x}](x^2-4)-xe^{3x}(2x)}{(x^2-4)^2}$$

12. $\arctan\sqrt{\frac{1-x}{1+x}}$

$$\frac{1}{\left(\frac{1-x}{1+x}\right)+1}\cdot\frac{1}{2\sqrt{\frac{1-x}{1+x}}}\cdot\frac{(-1)(1+x)-(1-x)(1)}{(1+x)^2} = \frac{1+x}{2}\cdot\frac{\sqrt{1+x}}{2\sqrt{1-x}}\cdot\frac{-2}{(1+x)^2}$$

$$= \frac{-1}{2\sqrt{(1-x)(1+x)}}$$

13. $\cos 3x\left(\frac{x+1}{e^x-1}\right)^7$

$$-3\sin 3x\left(\frac{x+1}{e^x-1}\right)^7 + 7\cos 3x\left(\frac{x+1}{e^x-1}\right)^6\cdot\frac{(1)(e^x-1)-(x+1)(e^x)}{(e^x-1)^2}$$

$$-3\sin 3x\left(\frac{x+1}{e^x-1}\right)^7 - 7\cos 3x\left(\frac{x+1}{e^x-1}\right)^6\cdot\frac{1+xe^x}{(e^x-1)^2}$$

Show that the following rule is true using the product or quotient rule as applicable.

14. Show: $\frac{d}{dx}\cot x = -\csc^2 x$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x\cdot\sin x - \cos x\cdot\cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

15. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(3) = -2$, $f'(3) = 5$, $g(3) = 3$, and $g'(3) = -1$. Compute the following derivatives:

a. $(fg)'(3) = f'(3)g(3)+f(3)g'(3) = 5\cdot 3+(-2)(-1) = 17$

b. $\left(\frac{f}{g}\right)'(3) = \frac{f'(3)g(3)-f(3)g'(3)}{[g(3)]^2} = \frac{5\cdot 3-(-2)(-1)}{3^2} = 13/9$

c. $\frac{d}{dx}[x^2f(x)]\Big|_{x=3} = 2xf(x)+x^2f'(x)\Big|_{x=3} = 6f(3)+9f'(3) = 6(-2)+9(5) = 33$

d. $\frac{d}{dx}\left[\frac{xf(x)+g(x)}{xg(x)}\right]\Big|_{x=3} = \frac{d}{dx}\left[\frac{f(x)}{g(x)}+\frac{1}{x}\right]\Big|_{x=3} = \left(\frac{f}{g}\right)'(3) - \frac{1}{x^2}\Big|_{x=3} = 13/9 - \frac{1}{9} = 4/3$