Key

Homework Set 7

Sect 3.3: Logarithmic Differentiation

Use logarithm rules or properties to simplify the following functions.

1. $\ln\left(x\sqrt{x^2-1}\right)$

$$= \ln x + \frac{1}{2}\ln(x^2-1)$$
$$= \ln x + \frac{1}{2}\ln(x-1) + \frac{1}{2}\ln(x+1)$$

2. $\ln\left(\frac{x^5e^{-x}}{\cos(x)}\right)$

$$= \ln x^5 + \ln e^{-x} - \ln(\cos x)$$
$$= 5\ln x - x - \ln(\cos x)$$

Use Logarithmic differentiation to compute $\frac{dy}{dx}$.

3. $y = (x^2+5)^3(x^2-1)^7$

$$\ln y = \ln(x^2+5)^3(x^2-1)^7$$
$$\ln y = 3\ln(x^2+5) + 7\ln(x+1) + 7\ln(x-1)$$
$$\frac{y'}{y} = 3\cdot\frac{2x}{x^2+5} + \frac{7}{x+1} + \frac{7}{x-1}$$
$$\rightarrow y' = \left[\frac{6x}{x^2+5} + \frac{7}{x+1} + \frac{7}{x-1}\right](x^2+5)^3(x^2-1)^7$$

4. $y = \sqrt{x}\cdot e^{3x^4}(x-2)^6$

$$\ln y = \frac{1}{2}\ln x + 3x^4 + 6\ln(x-2)$$
$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{1}{x} + 12x^3 + \frac{6}{x-2}$$
$$y' = \left[\frac{1}{2x} + 12x^3 + \frac{6}{x-2}\right]\sqrt{x}\,e^{3x^4}(x-2)^6$$

5. $y = \frac{x\sin^2 x}{x^2+x+2}$

$$\ln y = \ln x + 2\ln(\sin x) - \ln(x^2+x+2)$$
$$\frac{y'}{y} = \frac{1}{x} + 2\cdot\frac{\cos x}{\sin x} - \frac{2x+1}{x^2+x+2}$$
$$y' = \left[\frac{1}{x} + 2\cot x - \frac{2x+1}{x^2+x+2}\right]\frac{x\sin^2 x}{x^2+x+2}$$
$$\text{OR } = \frac{\sin^2 x}{x^2+x+2} + \frac{2x\sin x\cos x}{x^2+x+2} - \frac{(2x+1)x\sin^2 x}{(x^2+x+2)^2}$$

6. $y = \sqrt[3]{\dfrac{x+1}{x^4-1}}$

$$\ln y = \frac{1}{3}\ln(x+1) - \frac{1}{3}\ln(x^4-1) = \cancel{\frac{1}{3}\ln(x+1)} - \frac{1}{3}\ln(x^2+1) - \cancel{\frac{1}{3}\ln(x+1)} - \frac{1}{3}\ln(x-1)$$

$$\ln y = -\frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(x-1)$$

$$\frac{y'}{y} = -\frac{1}{3}\cdot\frac{2x}{x^2+1} - \frac{1}{3}\cdot\frac{1}{x-1}$$

$$y' = -\frac{1}{3}\left[\frac{2x}{x^2+1} + \frac{1}{x-1}\right]\sqrt[3]{\frac{x+1}{x^4-1}}$$

7. $y = (\sqrt{x})^x$

$$\ln y = x\ln\sqrt{x}$$

$$\ln y = \frac{1}{2}x\ln x$$

$$\frac{y'}{y} = \frac{1}{2}\ln x + \frac{1}{2}x\cdot\frac{1}{x}$$

$$y' = \left[\frac{1}{2} + \frac{1}{2}\ln x\right]\sqrt{x^x}$$

8. $y = (\tan x)^x$

$$\ln y = x\ln(\tan x)$$

$$\frac{y'}{y} = \ln(\tan x) + x\cdot\frac{\sec^2 x}{\tan x}$$

$$y' = \left[\ln(\tan x) + x\sec x\csc x\right](\tan x)^x$$

9. $y = (\ln x)^{1/x}$

$$\ln y = \frac{1}{x}\ln(\ln x)$$

$$\frac{y'}{y} = \frac{-1}{x^2}\ln(\ln x) + \frac{1}{x}\cdot\frac{1/x}{\ln x}$$

$$y' = \left[-\frac{\ln(\ln x)}{x^2} + \frac{1}{x^2\ln x}\right](\ln x)^{1/x}$$

10. $x^y = y^x$

$$\ln x^y = \ln y^x$$

$$y\ln x = x\ln y$$

$$y'\ln x + \frac{y}{x} = \ln y + \frac{x}{y}\cdot y'$$

$$y'\left[\ln x - \frac{x}{y}\right] = \ln y - \frac{y}{x}$$

$$y' = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}} = \frac{xy\ln y - y^2}{xy\ln x - x^2}$$