Key

Homework Set 9

2.8: Differentials

For questions 1 – 4, compute the differential of the following functions.

1. $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$dy = \frac{dx}{2\sqrt{x}}$$

2. $y = \sqrt{u}e^{-u}$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}}e^{-u} + \sqrt{u}\cdot(-e^{-u}) = \frac{1}{2e^{u}\sqrt{u}} - \frac{\sqrt{u}}{e^{u}}$$

$$dy = \left(\frac{1}{2\sqrt{u}} - \sqrt{u}\right)\frac{du}{e^{u}}$$

3. $y = \arctan(\ln t)$

$$\frac{dy}{dt} = \frac{1/t}{(\ln t)^2+1} = \frac{1}{t(\ln t)^2+t}$$

$$dy = \frac{dt}{t(\ln t)^2+t}$$

4. $y = x^5 - 4x^3 + x^2 + 8$

$$\frac{dy}{dx} = 5x^4 - 12x^2 + 2x$$

$$dy = (5x^4 - 12x^2 + 2x)\,dx$$

5. The surface area of a sphere is given by $A = \frac{4}{3}\pi r^2$. Find dr when $r = 3$.

$$\frac{dA}{dr} = \frac{4}{3}\pi \cdot 2r$$

$$dA = \frac{8\pi}{3} r\,dr \quad \text{if } r=3$$

$$dA = 8\pi \cdot dr$$

$$\frac{dA}{8\pi} = dr$$