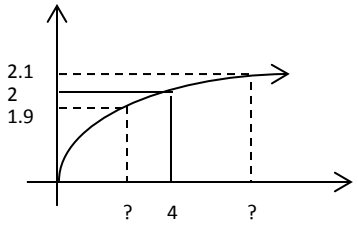
Homework Set 16

The Formal Definition of a Limit

(sections 1.3 & 1.6)

1. State the formal $\epsilon - \delta$ definition of a limit. (Hint: there are three versions.)

2. To illustrate $\lim_{x\to 4}\sqrt{x} = 2$, use the graph (below) of $f(x) = \sqrt{x}$ to find a number δ such that $|x - 4| < \delta$ guarantees that $\left|\sqrt{x} - 2\right| < 0.1$



3. Illustrate the formal $\epsilon - \delta$ definition of a limit applied to $\lim_{x\to 4}(3x - 5) = 7$ with a diagram (see figure 15 with example 9 in the textbook). Find a number δ such that $|x - 4| < \delta$ guarantees that $|f(x) - 7| < \frac{1}{4}$

4. Use the formal $\epsilon - \delta$ definition of a limit to show that $\lim_{x\to 4}(3x-5) = 7$.

5. To illustrate $\lim_{x\to\infty}\frac{2x}{x-1} = 2$, find a number N such that $x > N$ guarantees that $\left|\frac{2x}{x-1} - 2\right| < \frac{1}{100}$ (hint: you may want to look at the graph of $f(x) = \frac{2x}{x-1}$). What is the smallest x value such that $f(x)$ is within two decimal places of the limit?

6. Use the formal $\epsilon - \delta$ definition of a limit to show that $\lim_{x\to\infty}\frac{2x}{x-1} = 2$.