Project 1

Derivative Rules

(sect 2.3 - 2.6, 3.2 - 3.3, 3.5)

Part I: Derivatives

Calculate the first derivative of the given function. Give your answer in simplest form.

1.
$$y = -27.8$$

2.
$$f(x) = x^{131}$$

3.
$$g(x) = \frac{3}{x^8}$$

4.
$$y = -t^5 - 3t^2 + 4t + \cot 5t$$

5.
$$h(r) = e^{-r/6}$$

6.
$$y = \ln(x^3 + x + 2 - \cos x)$$

7.
$$y = 2 \ln \sec u$$

8.
$$g(t) = t^7 \ln 6t^4$$

9.
$$f(\theta) = 7e^{\sin\theta} \tan\theta$$

$$10. y = \ln \sin e^{5x}$$

Part II: Basic Derivatives

Find the derivative of each of the following functions by using one of the basic derivative rules (ie: don't use the product, quotient, chain, shifting, or scale rules).

11.
$$f(x) = 5^x - x^5 + 5\cos x + 5e^5$$

12.
$$g(t) = 7e^t + (t+1)^2 - (2t)^7$$

13.
$$k(x) = \frac{x^6 - 4x^5\sqrt{x} + 1}{x^5}$$

14.
$$y = \sqrt{x} \left(x^3 - 2\sqrt{x} + \frac{1}{x^2 \sqrt{x}} - 1 \right)$$

Part III: The Chain Rule

Find the derivative of each of the following functions. Use the Chain Rule, Shifting Rule, or Scale Rule as applicable.

15.
$$f(r) = 2e^{r^4 - 7r}$$

16.
$$y = (x^8 + 5x^3 - 4x + 7)^{15}$$

17.
$$v = \sqrt[3]{\ln x} + \ln \sqrt[3]{x}$$

18.
$$g(\theta) = \arctan(\theta^3 + 1) + 3$$

Part IV: The Product and Quotient Rules

Find the derivative of each of the following functions using either the product or quotient rules.

19.
$$y = \frac{x+2e^x}{x^2-5}$$

20.
$$g(t) = e^{-t}(t - \sin t)$$

21.
$$f(x) = \frac{\tan(3x)}{e^{3x}}$$

22.
$$y = \sqrt{1 - x^2} \cdot \arccos x$$

Part V: Derivative Rules (using several rules in the same question)

Find the derivative of each of the following functions using either the product, quotient, or chain rules or a combination of the rules.

$$23. f(x) = \left(\frac{x}{4-x}\right)^{-5}$$

24.
$$y = \arctan(x) \cdot \left(\frac{x+1}{x^2+1}\right)^3$$

25.
$$y = \frac{(1-x^5)^3}{x \sin x}$$

Part VI: Implicit Differentiation and Logarithmic Differentiation

Use either implicit or logarithmic differentiation to compute $\frac{dy}{dx}$.

26.
$$y = (x+3)^{x^2}$$

27.
$$y = \frac{e^{x^2} \tan^3 x}{\sqrt{x^2 + 1}}$$

$$28. \sqrt{y} + \sqrt{x} = 2xy$$

29.
$$e^y = 2x^2 + y^3 - \sin y$$

Part VII: Theory Question

30. Show that

$$\frac{d}{dx} \left(\frac{1}{2} \arctan(x) + \frac{1}{4} \ln \left(\frac{(x+1)^2}{x^2 + 1} \right) \right) = \frac{1}{(x+1)(x^2 + 1)}$$