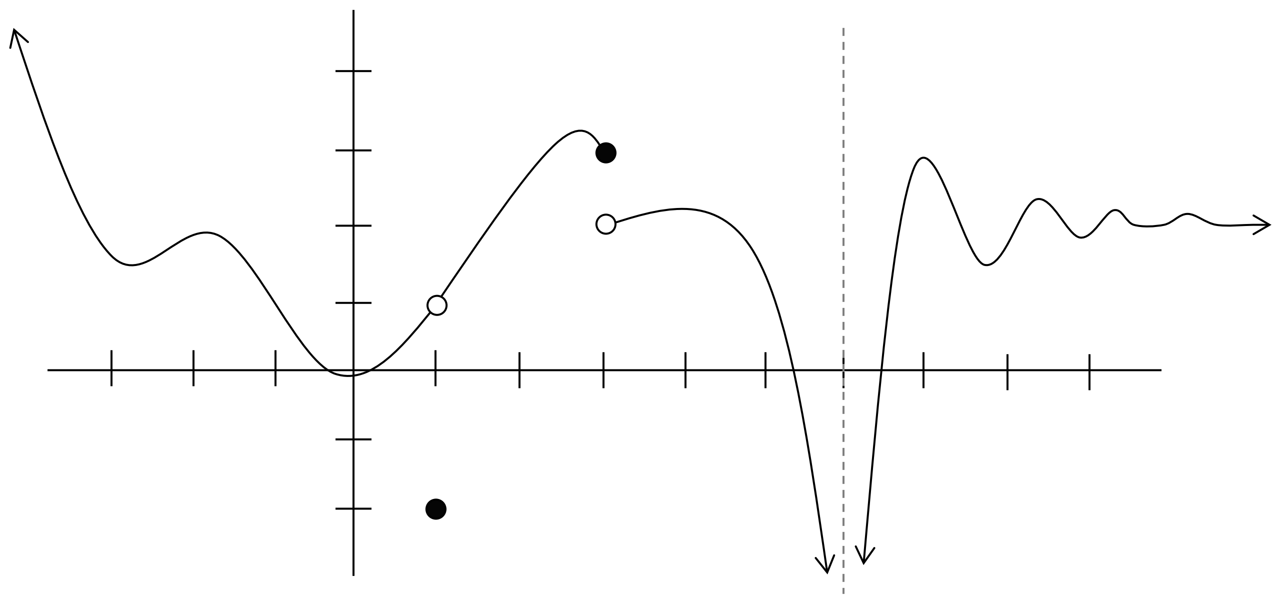
Project 2

Limits

(sect 1.3 – 1.6, 3.1 – 3.2, 3.7)

Part I: Evaluating Limits

1. Let $f(x)$ be the function whose graph is shown. Use this information to answer the questions below. You may assume that each hatch mark represents one unit.



$\lim_{x\to-\infty} f(x) =$

$\lim_{x\to-1} f(x) =$

$f(1) =$

$\lim_{x\to1^-} f(x) =$

$\lim_{x\to1^+} f(x) =$

$\lim_{x\to1} f(x) =$

$\lim_{x\to3^-} f(x) =$

$\lim_{x\to3^+} f(x) =$

$\lim_{x\to3} f(x) =$

$f(3) =$

$\lim_{x\to6} f(x) =$

$\lim_{x\to\infty} f(x) =$

Where is the function $f(x)$ continuous?

2. Compute $\lim_{x\to0^+}(1-3x)^{1/x}$ using the two methods below.
    a. Compute the above limit using a chart. Be sure to choose enough x-values so that your answer is accurate to at least 4 decimal places.

$\boldsymbol{x}$	$\boldsymbol{(1-3x)^{1/x}}$

$$\lim_{x\to0^+}(1-3x)^{1/x} =$$

    b. Compute the above limit using an appropriate method. (Do not plug in values.)

    c. Do your answers in part (b) and (c) agree? Is this expected? Why or why not?

3. Use the Squeeze Theorem to compute the following limit.

$$\lim_{x \to 0} \sqrt{x^2 + x} \cdot \cos^2\left(\frac{1}{x}\right)$$

$$\_\_\_\_\_\_\_\_\_\_ \leq \sqrt{x^2 + x} \cdot \cos^2\left(\frac{1}{x}\right) \leq \_\_\_\_\_\_\_\_\_\_$$

Part II: Using Limit Laws

4. Evaluate the following limit using the limit laws. Be sure to indicate which limit law you used at each step (either by name or number). Note, if you just plug in $x = 3$, you will get a zero on this question.

$$\lim_{x \to 3} \frac{x\left(x^2 + 2\sqrt{x} - 5\right)}{x^3 - x^2 - 7}$$

Part III: Continuous Functions and Limits

5. Determine where $f(x) = \frac{\sqrt{9-x^2}}{x^2-4}$ is continuous. Then compute the limit as $x \to 0$ using properties of continuity.

6. What should k equal in order to make the piecewise function $f(x)$ continuous?

$$f(x) = \begin{cases} \dfrac{\tan(5x)}{x} & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$$

Part IV: Using Limit Rules and Methods

Compute the following limits using a method we learned in class.

7.

$$\lim_{x \to 1} \left(\frac{x^2 - 1}{x^2 - x} \right)$$

8.

$$\lim_{x \to 3} \left(\frac{\frac{x}{3} - \frac{3}{x}}{\frac{1}{3} - \frac{1}{x}} \right)$$

9.

$$\lim_{x \to 0} \frac{(x-9)^2 - 81}{2x}$$

10.

$$\lim_{x \to 5} \frac{\sqrt{9-x} - \sqrt{x-1}}{x-5}$$

Part V: Limits with Exponential and Inverse Functions

Compute the following limits using the limit properties of exponential functions, logs, and inverse trig functions. Do not use L'Hopital's Rule to find the limit.

11.

$$\lim_{x \to -\infty} \left(\frac{2}{3}\right)^x =$$

12.

$$\lim_{x \to \infty} e^{-x^2} =$$

13.

$$\lim_{x \to 0} \ln(\cos(x)) =$$

14.

$$\lim_{x \to \infty} \arctan\left(\frac{2 - x - 3x^2}{1 + 3x^2}\right) =$$

Part VI: Limits as x Approaches Infinity

Compute the following limits using a method we learned in class.

15.

$$\lim_{x \to -\infty} \frac{7x^3 + 3x^2 - 12x + 11}{5x^3 - 6x + 31}$$

16.

$$\lim_{x \to \infty} \frac{4x^2 - 7x - 3}{x^3 + x^2 + 4}$$

17.

$$\lim_{x \to -\infty} \frac{4x^2 - 3x + 2}{4 - x}$$

18.

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 4x} - \sqrt{x^2 - x} \right)$$

Part VII: L'Hopital's Rule

Use L'Hopital's Rule to compute the limit.

19.

$$\lim_{x \to 1} \frac{x^9 - 1}{x^3 - 1}$$

20.

$$\lim_{x \to 0} \cot 3x \sin 6x$$

21.

$$\lim_{x \to \infty} \frac{e^{2x}}{x^2}$$

22.

$$\lim_{x \to \infty} \left(1 - \frac{1}{2x}\right)^{8x}$$

23.

$$\lim_{x \to \infty} x^{(ln2)/(1+lnx)}$$

Part VIII: Concept Questions

Questions 24 and 25 are extra credit questions. Either of these questions you solve correctly will give you one point of extra credit. Be careful when you work through these questions. Since these questions ask you to "show" a formula (technically called proof questions), you have to write down all of the steps to get credit.

24. For what values of a and b is the following equation true?

$$\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$$

25. For the function $f(x)=x^3-5$, show that

$$\lim_{h\to 0}\frac{f(x+h)-2f(x)+f(x-h)}{h^2}=f''(x)$$