Project 4

The Derivative

(sect 2.1 – 2.2, 2.8, 4.1 – 4.4)

Part I: Tangent Lines and the Definition of the Derivative.

1. Find the derivative of $f(x) = \frac{4}{x}$ using the limit definition of the derivative.
   a. Find the derivative of $f(x)$ using the limit definition of the derivative.

   b. Find $f'(x)$ using derivative rules.

   c. Are your answers from part (a) and part (b) the same? Should the two answers be the same? Why or why not.

2. For the following questions, use $f(x) = 1 - 3x + 5x^2$.
   a. Find the derivative of $f(x)$ using the limit definition of the derivative.

   b. Find $f'(x)$ using derivative rules. Is your answer the same as from part (a)? Should the two answers be the same? Why or why not.

   c. Find the equation of the line tangent to $f(x)$ at the point $x = -1$.

3. Find equation of the line tangent to the function $y = e^x + 3$ at $x = \ln 2$.

Part II: Properties of a Function and Curve Sketching.

4. Sketch the graph of a function $f(x)$ which has the following properties:

$$f(2) = f(4) = 0$$
$$f(3) \ is \ defined$$
$$f'(x) < 0 \ \ if \ x < 3$$
$$f'(3) \ is \ undefined$$
$$f'(x) > 0 \ \ if \ x > 3$$
$$f''(x) < 0 \ \ for \ x \neq 3$$

5. Consider the function $f(x) = (x-1)^2(x+2)$ on the interval $[-3,2]$
   a. Where is the function increasing? Decreasing?

   b. What are the maximum(s) and minimum(s) of the function?

   c. Where is the function concave up? Concave down?

   d. What are the inflection point(s) of $f(x)$?

   e. Use all of the information you found above to sketch the graph of the function. Be sure to label all the important parts.

6. Consider the function $f(x) = x + \frac{4}{x}$

   a. Where is the function increasing? Decreasing?

   b. What are the maximum(s) and minimum(s) of the function?

   c. Where is the function concave up? Concave down?

   d. What are the inflection point(s) of $f(x)$?

   e. Use all of the information you found above to sketch the graph of the function. (Hint: look at the end behavior and the asymptotes of the function.)

Part III: Using the Derivative.

7. Use linear approximation to estimate the value of $\sqrt{99.8}$

8. Suppose $f(x)$ is continuous on the interval [2, 5]. If $f(2) = -1$ and $-3 \leq f'(x) \leq 1$ for all x in [2, 5]. Use the Mean Value Theorem to find the largest and smallest values that $f(5)$ can obtain.

$$\_\_\_\_\_\_\_ \leq f(5) \leq \_\_\_\_\_\_\_$$

9. At 2:00 pm, a car's speedometer reads 30 mph. At 2:10 pm, it reads 50 mph. Show that at some time between 2:00 pm and 2:10 pm the acceleration is exactly 120 mi/hr^2.

10. You are asked to design the first ascent and drop for a new roller coaster. By studying photographs of your favorite roller coasters, you decide to make the slope of the ascent 0.8 and the slope of the drop -1.6. You decide to connect these two straight stretches $y = L_1(x)$ and $y = L_2(x)$ with part of a parabola $y = f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in feet. For the track to be smooth there cannot be abrupt changes in direction, so you want the linear segments L_1 and L_2 to be tangent to the parabola at the transition points P and Q. To simplify the equations, let P be the origin.

   a. Suppose the horizontal distance between P and Q is 100 ft. Write equations in a, b, and c that will ensure that the track is smooth at the transition points. Use these equations to solve for a, b, and c to find a formula for $f(x)$.

   b. Find the difference in elevation between P and Q.

   c. Find the equations for the straight lines L_1 and L_2.

   d. Plot L_1, f, and L_2 to verify graphically that the transition points are smooth.