Project 5

Applications

(sect 2.7 – 2.8, 3.4, 4.5 – 4.7)

1. Consider $\sqrt[4]{15}$ for this question.
   a. Use linear approximation to estimate the value of $\sqrt[4]{15}$.

   b. Use Newton's method to estimate the value of $\sqrt[4]{15}$ to 6 decimal places.

      $f(x) =$ $\qquad\qquad$ $f'(x) =$

      Pick x_1 by choosing an integer close to the root (ie: this is your first guess).

      $x_1 =$

      $x_2 =$

      $x_3 =$

      $x_4 =$

      $\sqrt[3]{15} =$

   c. Which of these two methods is quicker to use to compute $\sqrt[4]{15}$? Which is more accurate? How would you determine when to choose to use each of these methods?

2. Water is leaking out of an inverted conical tank at a rate of $0.01\ m^3/sec$ at the same time that water is being pumped into the tank at a constant rate. The tank has a height of $6\ m$ and the diameter at the top is $4\ m$. If the water level is rising at a rate of $0.25\ m/sec$ when the height of the water is at $2\ m$, find the rate at which water is being pumped into the tank. (Hint: $\frac{dV}{dt} = (rate\ of\ water\ being\ pumped\ in) - (rate\ of\ leak)$)

3. When blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel. (This is known as Poiseuille's Law.) A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.
    a. Give an expression for F in terms of R.

    b. Show that the relative change in F is about four times the relative change in R.

    c. How will a 5% increase in the radius affect the flow of blood?

4. Find the function $f(x)$ if $f''(x) = 2 + e^x - sinx$; $f(0) = 0$, $f'(0) = 5$.

5. A bacteria culture grows with a constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 75,000.
    a. Find the initial population.

    b. Find an expression for the population after t hours.

    c. Find the number of cells after 5 hours.

    d. Find the rate of growth after 5 hours.

    e. When will the population reach 200,000?

6. The natural growth model is not the only model used to predict the growth (or decay) of a population. The logistic growth model is typically used when the population can only get to be so large. It is described by $P(t) = \frac{k}{1+Ae^{-rt}}$. Suppose the logistic growth function $P(t) = \frac{100{,}000}{1+5000e^{-t}}$ describes the number of people, $P(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.
    a. How many people became ill with the flu when the epidemic began?

    b. How many people were ill by the end of the fourth week?

    c. What is the rate at which people are being ill by the end of the fourth week?

    d. What is the limiting size of the population that becomes ill?

7. The range R of a projectile fired with an initial velocity of v_0 at an angle θ relative to the horizontal is $R = \frac{v_0^2 \sin 2\theta}{g}$ where g is the acceleration due to gravity. Find the angle θ such that the range of the projectile is maximized. (Hint: g and v_0 are constants.)

8. Find two positive numbers where the sum of the first and twice the second is 100, and the product is a maximum.

9. A farmer plans to fence a rectangular pasture adjacent to a river. He plans to subdivide this area into two fields with the one additional fence dividing the field in half running perpendicular to the river. The two fields together need to enclose 1,080,000 square meters in order to provide enough grass for the animals. What dimensions would require the least amount of fencing if no fencing is needed along the river?

10. A wooden beam has a rectangular cross section of height h and width w (see figure). The strength S of the beam is directly proportional to the width and the square of the height. What are the dimensions of the strongest beam that can be cut from a round log of diameter of 24 inches? (hint: $S = kh^2w$, where k is the proportionality constant.)