Chapter 3 Review
Sec 3.1 and 3.2

Exponential, Logarithmic and Inverse Functions

I. Review of Inverse Functions

A. Identifying One-to-One Functions

A function $f(x)$ is one-to-one if every element in the range corresponds to only one element in the domain.

If $f(a)=f(b)$ then $a=b$ or if $a \neq b$ then $f(a) \neq f(b)$

Horizontal Line Test: If there is NO horizontal line that intersects the graph more than once, then the function is one-to-one.

Example: Determine whether each function is one-to-one.

1.) $f(x)=x^2+2$

2.) $g(x)=\dfrac{x}{x-2}$

B. Inverse Functions

Let $f(x)$ be a function that is one-to-one and that goes through the point (a, b)

- Then $f^{-1}(x)$ is the inverse of $f(x)$
- $(f \circ f^{-1})(x)=x$
- $f^{-1}(x)$ will go through the point (b, a)
- The domain of $f(x)$ = the range of $f^{-1}(x)$
- The domain of $f^{-1}(x)$ = the range of $f(x)$

C. Finding Inverse Functions

Steps:
1. Test to see whether the function is one-to-one
2. Replace $f(x)$ with y
3. Interchange x and y
4. Solve equation for y
5. Replace y with $f^{-1}(x)$

Example: Verify that the functions are inverse of each other

1.) $f(x)=\sqrt[3]{x+4}$ and $g(x)=x^3-4$

Example: Find the inverse for each of the following

1.) $k(x) = (x-1)^3$

2.) $g(x) = \dfrac{2x-3}{x+1}$

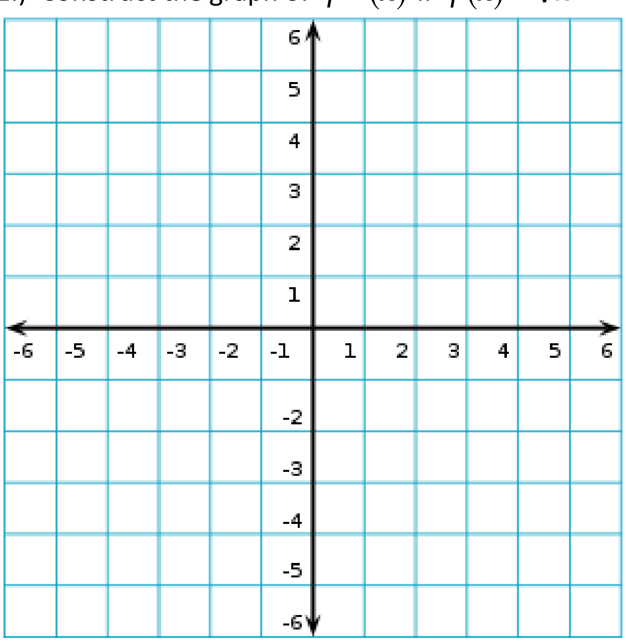
3.) $h(x) = x^2 + 3$

D. Graphs of Inverse Functions

> The graph of $f^{-1}(x)$ can be constructed by mirroring the graph of $f(x)$ over the line $y = x$

Examples:

1.) Construct the graph of $f^{-1}(x)$ if $f(x) = \sqrt{x}$



2.) The following are points on the graph of $f(x)$: $(2, 10)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(3, 6)$

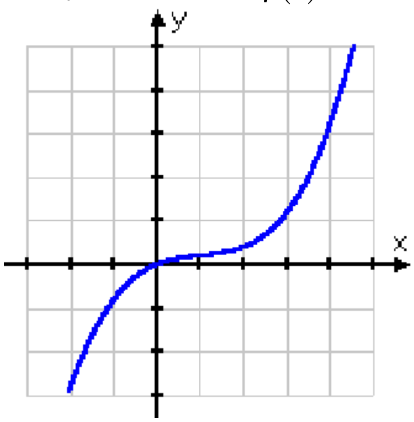
Find at least 5 points on the graph of $f^{-1}(x)$

D. Domain and Range of Inverse Functions

The domain of $f(x)$ = the range of $f^{-1}(x)$, and

The domain of $f^{-1}(x)$ = the range of $f(x)$

Example: A function $f(x)$ has the following graph. Find the domain and Range of the inverse function $f^{-1}(x)$



II. Review of Exponential and Logarithmic Functions

A. Exponential Functions

DEFN: An exponential function is a function in the form $f(x) = a^x$. (i.e. the variable x is in the exponent)

Example: Find x for each of the following:

1.) $4 = 2^x$

2.) $27 = 3^x$

3.) $2 = 4^x$

B. Logarithmic Functions

I. Logarithmic Functions

A logarithm is a function that helps us to solve a quadratic function / logarithms allow us to isolate the variable in a quadratic function (and the other way around).

DEFN: A logarithmic function is a function in the form $f(x) = \log_a x$. (i.e. the variable x is in the expression)

$y = \log_b x$ "**y** is equal to log base **b** of **x**" - Here "**b**" is the BASE NUMBER and "**x**" is the VARIABLE.

$\log_b x = y$ means exactly the same thing as $b^y = x$

Examples: Write each equation in its equivalent form:

1.) $x = \log_2 16$

2.) y = $\log_6 216$

3.) $3 = \log_b 27$

4.) $8^y = 300$

II. Common Logarithmic Properties

1. $\log_b b = 1$ 2. $\log_b 1 = 0$ 3. $\log_b 0 = DNE$	4. $\log_b b^x = x$ 5. $b^{\log_b x} = x$ 6. $\log x = \log_{10} x$

Example: Simplify Each Expression

1.) $\log_2 2 =$

2.) $\log_6 1 =$

3.) $\log_4 4^x =$

4.) $\log_z z^{(x+y)} =$

5.) $8^{\log_8 (12y)} =$

6.) $\log 10 =$

III. The Natural Logarithm

DEFN: e is a number the equals approximately 2.718281828

$$\log_e x = \ln x$$

Example:

1.) $\log_e (3z) =$

2.) $\ln e =$

IV. Expansion Properties for Logarithms

1. $\log_b(M \cdot N) = \log_b M + \log_b N$ (Product Rule)
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ (Quotient Rule)
3. $\log_b(M^n) = n \cdot \log_b M$ (Power rule)

Example: Simplify the following:

1.) $\log_x(xz) =$

2.) $\log_3 27 =$

3.) $2^{\log_2(5y)} =$

4.) $\log 10^{(x+y)} =$

Expand the following logarithms

5.) $\log_b\left((xy)^3\right)$

6.) $\log_b\left(xy^3\right)$

7.) $\log_b\left(\frac{x^3 y}{z^2}\right)$

8.) $\ln\left(\sqrt{ex}\right)$

9.) $\ln\left(\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right)$

10.) $\log\left(\frac{100x(x+3)}{z(x-4)^3}\right)$

Write the following as single logarithms

11.) $3\log_b(z)+4\log_b(x)$

12.) $\frac{1}{2}\log(x+2)-\log(x)$

13.) $3\log_5 z-\frac{1}{2}\log_5(z+2)+2\log_5 y$

V. Change of Base Formula for Logarithms

$$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Example: Use your calculator to find:

1.) $\log_8 17$

2.) $\log_5 15$

C. Solving Exponential and Logarithmic Functions

I. Common Base Property for Exponential Functions

If $b^M = b^N$, then $M = N$

Example:

1.) Solve $3^{2x} = 3^{x-5}$

2.) Solve $8^x = 2^{x+4}$

II. "Exponentiating" (How to solve equations involving e and ln)

$$\ln b^x = x \cdot \ln b$$

$$\ln e^x = x$$

Example:

1.) Find x if: $e^{7x+3} = 5$

2.) Find x if: $2^x = 12$

III. Common Base Property for Logarithmic Functions

> If $\log_b M = \log_b N$, then $M = N$

Example: Solve $\ln(x+2) - \ln(4x+3) = \ln\left(\frac{1}{x}\right)$

IV. Solving for a variable in the exponent.

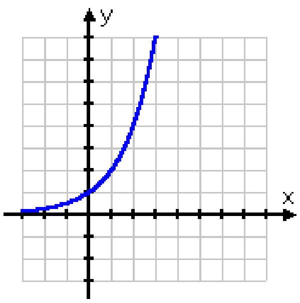
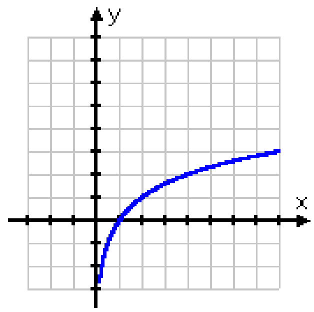
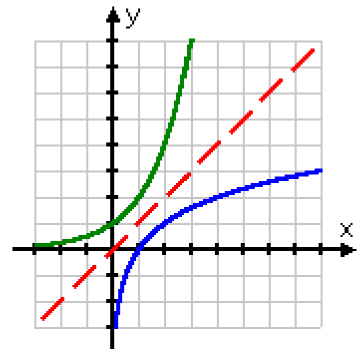
Example:

1.) $R = 25 \cdot e^{38 \cdot x}$

2.) $y = 8 \cdot e^{t \cdot x}$

D. Graphs of Exponential and Logarithmic Functions

I. Comparison of Logarithmic function graph to Exponential function graph

$y = 2^x$	$y = \log_2(x)$	Comparison of the two graphs, showing the inversion line in red.
		

If $f(x) = a^x$ and $g(x) = \log_a x$ then $f(x)$ and $g(x)$ are inverses of each other.

$f(x) = a^x$	*Domain*:	**All x**	(No Restrictions)	$(-\infty, \infty)$
	Range:	**y > 0**		$(0, \infty)$
$f(x) = \log_a x$	*Domain*:	**x > 0**		$(0, \infty)$
	Range:	**All x**	(No Restrictions)	$(-\infty, \infty)$

* Note: Since exponential and logarithmic functions (with the same variable and base number) and are inverses of each other, the domain of one is the range of the other and vice versa.

Example:
Find the domain and range of

1.) $f(x) = \log_5 x$

2.) $f(x) = \log_5(x-4) + 2$

3.) $f(x) = \ln(x^2 - 1)$