Sec 1.1 and 1.2 Functions: Review of Algebra and Trigonometry

A. Functions and Relations

DEFN **Relation**: A set of ordered pairs.
(x,y) $\rightarrow$ (domain, range)

DEFN **Function**: A correspondence from one set (the domain) to anther set (the range) such that each element in the domain corresponds to exactly one element in the range.

Example: Determine whether each of the following is an example of a function or not.

1.) $\{(1,1)(3,2)(5,3)\}$

2.) $\{(1,1)(2,4)(3,-5)(2,-4)\}$

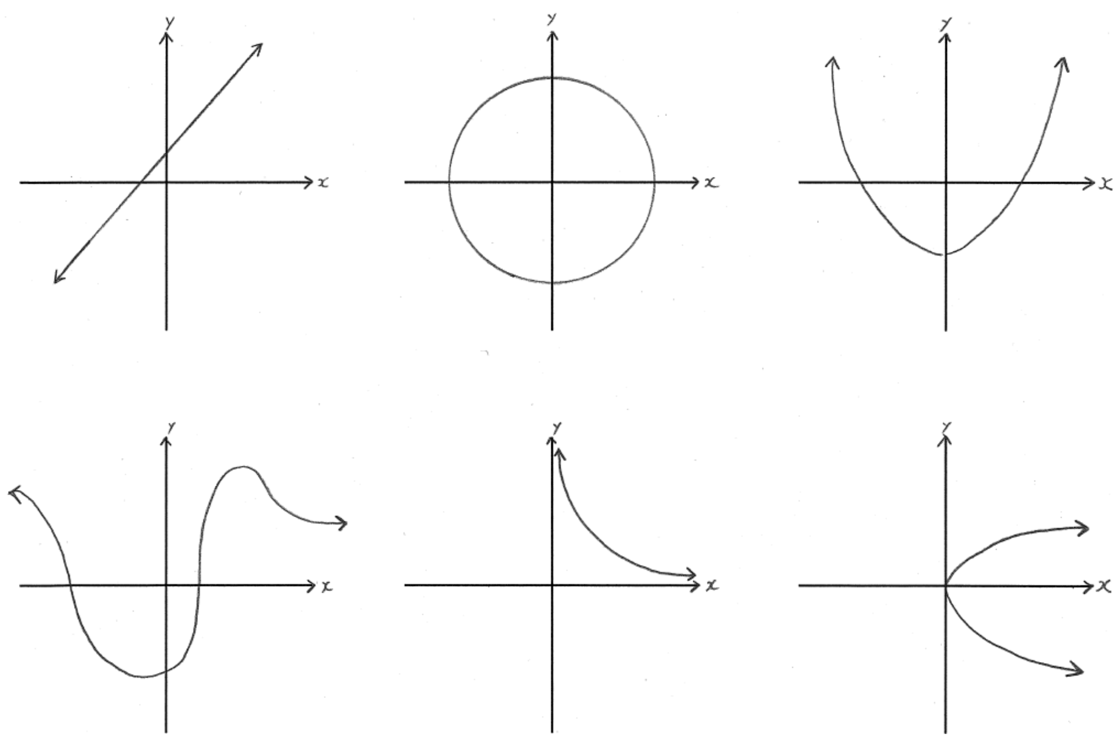
3.) $x^2+2y=0$

4.) $x^2+y^2=1$

<u>Vertical Line Test for Functions</u>

Vertical Line Test for Functions: If any vertical line intersects a graph more than once, then the graph is **<u>not</u>** a function.

Example: Determine whether each of the following is a function or not by the Vertical Line Test.



B. Domain and Range of a Function

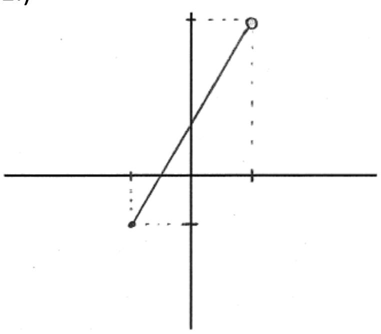
DEFN **Domain**: Input $\longrightarrow$ x-values (i.e. All of the values of x that I may plug into a function.)

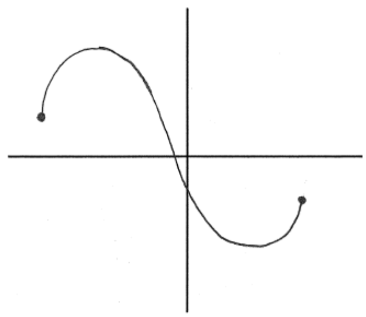
DEFN **Range**: Output $\longrightarrow$ y-values (i.e. All of the values of y that a function can attain)

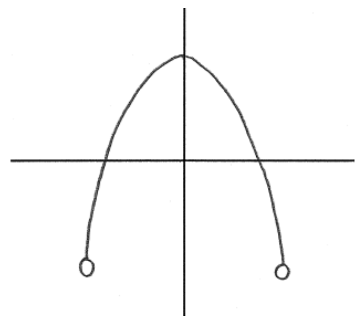
Function Notation: $f(x) = y$

Example: Give the domain and range (in interval notation) for each of the following

1.)







Domain:________________ Domain:________________ Domain:________________

Range: ________________ Range: ________________ Range: ________________

Example: Give the domain for each of the following (in interval notation and as an inequality)

2.) $g(x) = \sqrt{2-x}$

3.) $f(x) = \dfrac{x^2 - 4}{x+2}$

4.) $g(x) = \dfrac{3x}{\sqrt{x+5}}$

5.) $h(x) = \ln(6x-3)$

* The 3 functions for which we will most frequently have domain restrictions (in this course) are: **fractions (aka…rational functions)**, **radicals** and **logarithms**.

C. Linear Models

Definition

$$\textbf{Slope} = m = \frac{rise}{run} = \frac{\Delta\, y}{\Delta\, x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (Use when given two points to find slope)
Slope-Intercept Form	$y = mx + b$ (Use when given slope and y - intercept)
Point-Slope Form	$(y - y_1) = m \cdot (x - x_1)$ (Use when given one point and slope)
General Form	$A \cdot x + B \cdot y + C = 0$
Horizontal Line	$y = b$ (where b = constant)
Vertical Line	$x = c$ (where c = constant)

Parallel Lines

Two lines are parallel if and only if they have the same slope.

For two lines $y_1 = m_1 x + b_1$ and $y_2 = m_2 x + b_2$ we have $y_1 \parallel y_2 \iff m_1 = m_2$

Perpendicular lines

Two lines are perpendicular if and only if the product of their slope = -1.

For two lines $y_1 = m_1 x + b_1$ and $y_2 = m_2 x + b_2$ we have $y_1 \perp y_2 \iff m_1 = -\dfrac{1}{m_2}$

Examples: Find the equation of the line:

1.) that passes through point (0 , -3) with slope = -2

2.) that passes through points (3 , -2) and (4 , 5)

3.) that passes through point (0, 0) and is parallel to the line $y = 3x - 9$

4.) that passes through point (2 , -4) and is perpendicular to the line $y = \frac{1}{2}x + 3$

5.) Find the slope and y-intercept of the line $9x - 3y - 3 = 0$

D. Classes of Functions

1. Power Functions

For any real number m, a function in the form $f(x) = x^m$ is called a Power Function

2. Polynomials

Definition

A *polynomial function* is a function in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ where $a_0 \neq 0$ and n is a positive integer.

Examples: State whether each is a polynomial:

1.) $g(x) = x^2 + 5x + 6$

2.) $f(x) = x^3 + 7x - \sqrt{x}$

3.) $f(x) = \frac{x^2}{3} + 8x$

4.) $h(x) = x^{\frac{2}{3}} - x + 5$

5.) $f(x) = \frac{4}{x^2} + 6x - 1$

3. Rational Functions

A Rational Function is the quotient of two polynomial functions:

A Rational Function is a function of the form $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0}$

Asymptotes

An asymptote is an imaginary line that the graph of a function approaches as the function approaches a restricted number in the domain or as it approaches infinity.

I. Locating Vertical Asymptotes

If $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function, $p(x)$ and $q(x)$ have no common factors and n is a zero of $q(x)$, then the line $x = n$ is a vertical asymptote of the graph of $f(x)$.

II. Locating Horizontal Asymptotes

Let $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0}$

i. If n < m, then $y = 0$ is the horizontal asymptote ("Bottom Heavy")

ii. If n = m, then the line $y = \dfrac{a_n}{b_m}$ is the horizontal asymptote ("Equal Degree")

iii. If n > m, there is NO horizontal asymptote. (But there will be a slant/oblique asymptote) ("Top Heavy")

Examples: Find all vertical and horizontal asymptotes:

1.) $f(x) = \dfrac{15x}{3x^2 + 1}$

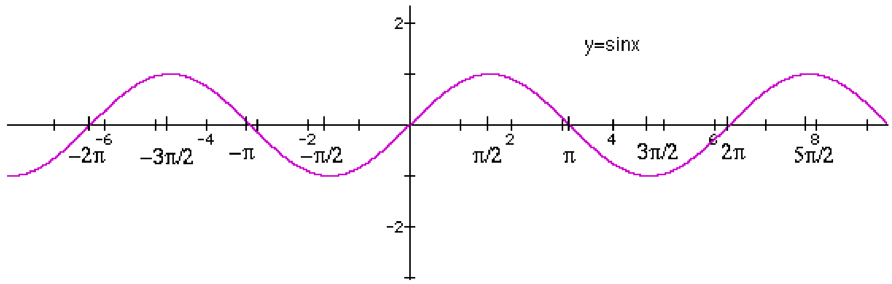
2.) $g(x) = \dfrac{15x^3}{3x^2 + 1}$

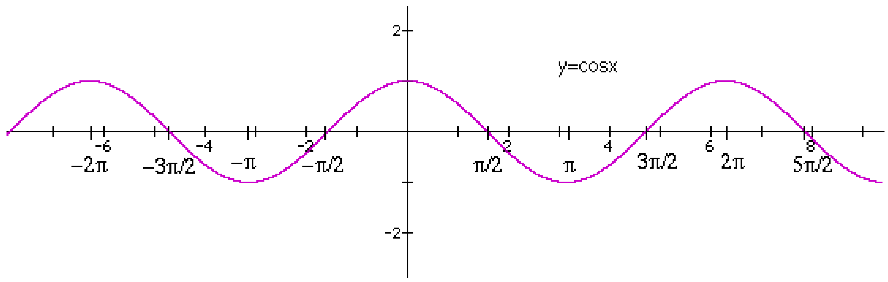
3.) $h(x) = \dfrac{-3x + 7}{5x - 2}$

4.) $k(x) = \dfrac{2x - x^2}{x^2 - 2x - 3}$

4. Trigonometric Functions

sin(x)	csc(x)
cos(x)	sec(x)
tan(x)	cot(x)





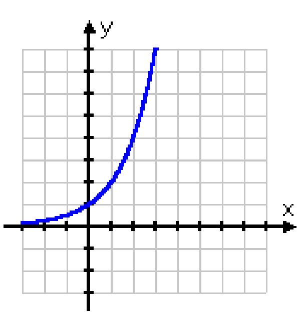
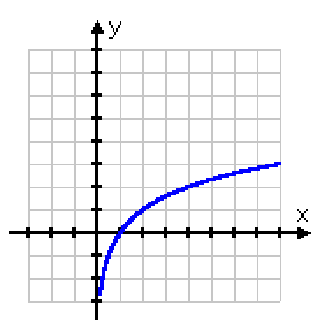
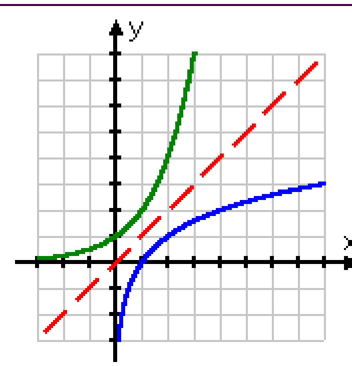
5. Exponential and Logarithmic Functions

DEFN: An exponential function is a function in the form $f(x) = a^x$. (i.e. the variable x is in the exponent)

DEFN: A logarithmic function is a function in the form $f(x) = \log_a x$. (i.e. the variable x is in the expression)

$y = \log_b x$ "**y** is equal to log base **b** of **x**" - Here "**b**" is the BASE NUMBER and "**x**" is the VARIABLE.

$\log_b x = y$ means exactly the same thing as $b^y = x$

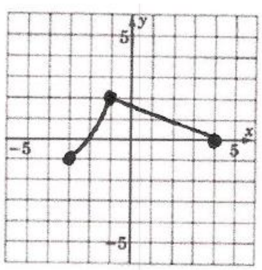
$y = 2^x$	$y = \log_2(x)$	Comparison of the two graphs, showing the inversion line in red.
		

E. Transformations of Functions

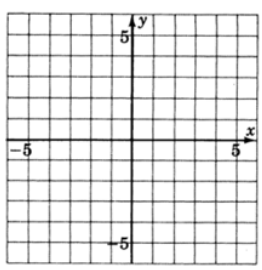
Vertical Shifts	$f(x)+C$	↑	Moves Graph **UP C** units
	$f(x)-C$	↓	Moves Graph **DOWN C** units
Horizontal Shifts	$f(x-C)$	→	Moves Graph RIGHT C units
	$f(x+C)$	←	Moves Graph LEFT C units
Vertical and Horizontal Reflections	$-f(x)$	↕	Flips Graph About x-axis
	$f(-x)$	↔	Flips Graph About y-axis
Vertical Stretching/ Compressing	$c \cdot f(x)$ for $c>1$	↑ ↓	Graph Vertically Stretches by a Factor of C
	$c \cdot f(x)$ for $0<c<1$	↓ ↑	Graph Vertically Shrinks by a Factor of C

Example: Use the given graph of ***f(x)*** to sketch each of the following.

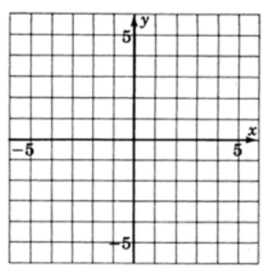
f(x)



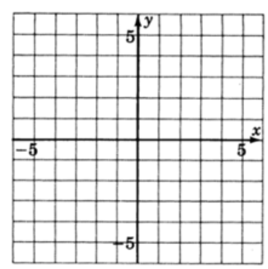
a. f(x) + 2



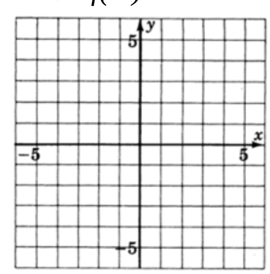
b. f(x + 2)



c. –f(x)



d. 2f(-x)

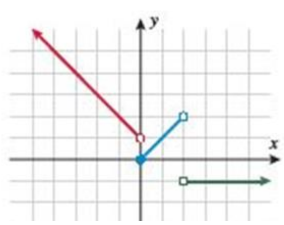


F. Combinations of Functions

1. Piecewise-Defined Functions

A Piecewise Function is a function that has specific (and different) definitions on specific intervals of x.

$$f(x)=\begin{cases} -x+1 & x<0 \\ x & 0\le x<2 \\ -1 & x>2 \end{cases}$$



Domain: ______________________________

Range: ______________________________

2. Sums, Differences, Products and Quotients of Functions

Sum	$(f+g)(x)=f(x)+g(x)$
Difference	$(f-g)(x)=f(x)-g(x)$
Product	$(f\cdot g)(x)=f(x)\cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

Example

1. If $f(x) = 2x+1$ *and* $g(x) = x^2 - 3$ find each of the following.

a. $f(4)$ *b.* $g(2x)$ *c.* $f(3x-4)$ *d.* $f(x) + g(x)$ *e.* $f(x)g(x)$

3. Composition of Functions

Notation	$(f\circ g)(x)=f(g(x))$

Example:

1.) For the functions $f(x)=\sqrt{x}$ and $g(x)=x+2$ find

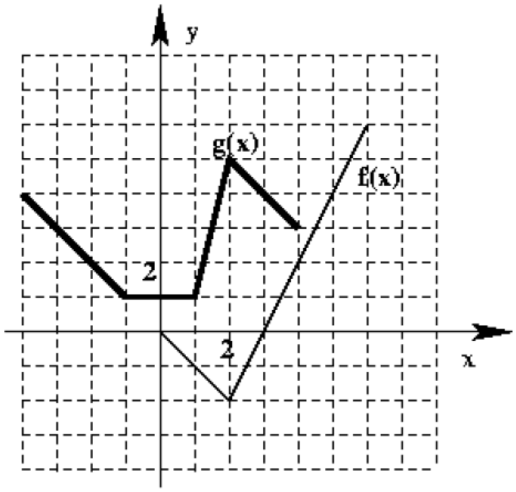
a.) $(f\circ g)(x)=$

b.) $(g\circ f)(x)=$

c.) $(f\circ g)(2)=$

d.) $(g\circ g)(x)=$

Example: For the functions f(x) and g(x) given in the graph find



a.) $(f \circ g)(2) =$

b.) $(f \circ g)(3) =$

c.) $(g \circ f)(2) =$

d.) $(g \circ f)(3) =$

e.) $(f \circ f)(4) =$

f.) $(g \circ g)(4) =$

G. Symmetry

Symmetry:	**Even** functions… f(x) = f(- x) …. Symmetric about the y-axis …. If (a,b) then (-a,b)
	Odd functions…f(- x) = - f(x) …. Symmetric about the origin …. If (a,b) then (-a,-b)

1. State whether the following functions are even, odd, or neither.

a. $f(x) = x^5 + 5x$

b. $f(x) = 1 - x^4$

c. $f(x) = 2x - x^2$

d. $f(x) = 2\sin(x)$

e. $f(x) = \cos(x) - 1$

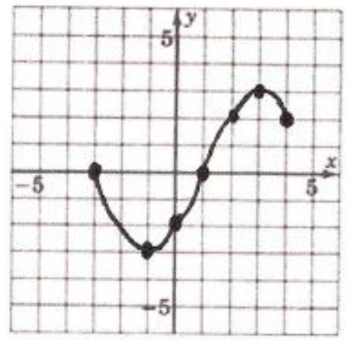
f. $f(x) = |x| - 3$

H. Function Properties

- **Increasing** functions rise from left to right	**Decreasing** functions fall from left to right
Positive functions are above the x-axis	**Negative** functions are below the x-axis

For all of these above, you use the x-values to state your answers!

1. Find each of the following using the given function.



a. $f(x) > 0$

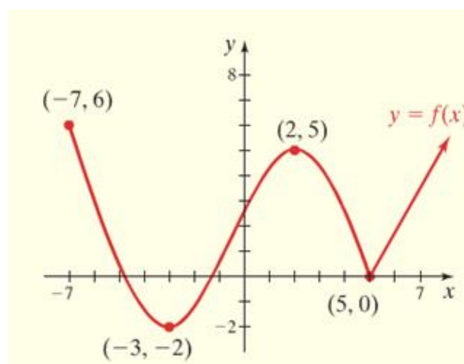
b. $f(x) \leq 0$

c. *increasing*

d. *decreasing*

e. *domain and range*

2. Find each of the following using the given function.



a. $f(x) > 0$

b. $f(x) \leq 0$

c. *increasing*

d. *decreasing*

e. *domain and range*