Sec 1.3 The Limit of a Function

A. Limits

DEFN:	$\lim_{x \to a} f(x) = L$	The limit of f(x) as x approaches a, equals L.
	(Where is the functions value headed as x is "on its way" to a?)	
	$\lim_{x \to a^-} f(x)$	The limit of f(x) as x approaches a from the LEFT
	$\lim_{x \to a^+} f(x)$	The limit of f(x) as x approaches a from the RIGHT

B. Techniques of Solving Limits

1. Evaluation - When possible (without violating domain rules) "plug it in".

Example:

1.) $\lim_{x \to 3} x^2 =$

2.) $\lim_{x \to 1} \frac{1}{x} =$

2. Factoring/Manipulation (then Evaluation) - Factor expressions and cancel any common terms.

Example:

1.) $\lim_{x \to 4} \frac{x-4}{x^2-16} =$

2.) $\lim_{x \to 3} \frac{x^2-3x}{x^2-2x-3} =$

3. Table - Set up a table as x approaches the limit from the left and from the right.

Example:

1.) $\lim_{x \to 0} \frac{\sin x}{x} =$

4. Graphing - Graph the function and inspect. (Warning: Your graphing calculator might not always indicate a hole or small discontinuity in a graph. Be sure to always check the domain for restrictions.)

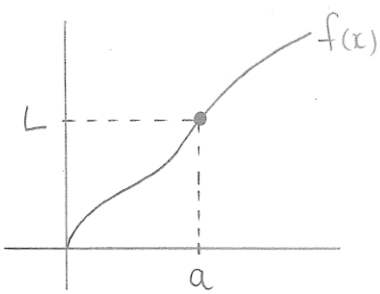
Example:

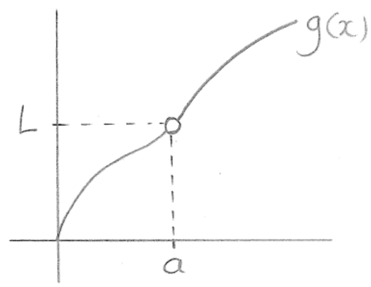
1.) $\lim\limits_{x\to 0} \dfrac{\sin x}{x} =$

More Examples:

1.) $\lim\limits_{x\to 0^+} \dfrac{1}{x} =$

2.)





$f(a)=$

$\lim\limits_{x\to a} f(x)=$

$g(a)=$

$\lim\limits_{x\to a} g(x)=$

3.) $f(x)=\begin{cases} 7-x & \text{if } x\le -4 \\ x & \text{if } -4<x\le 2 \\ (x-1)^2 & \text{if } x>2 \end{cases}$

$\lim\limits_{x\to -4^-} f(x)$

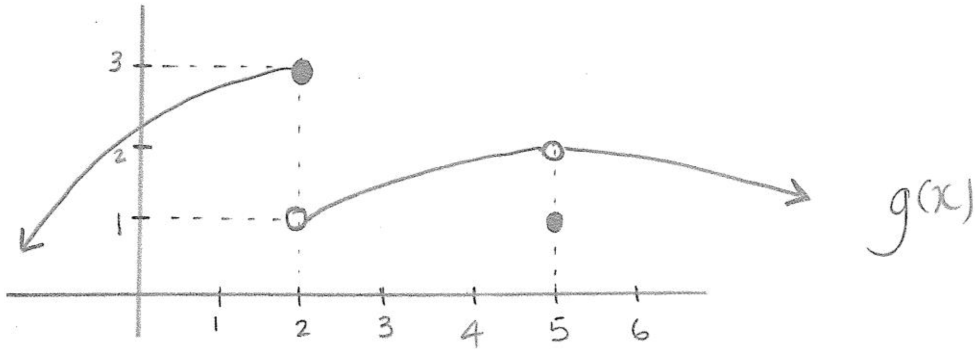
$\lim\limits_{x\to -4^+} f(x)$

$\lim\limits_{x\to -4} f(x)$

$\lim\limits_{x\to 2^-} f(x)$

$\lim\limits_{x\to 2^+} f(x)$

4.)



$g(2)=$ $\qquad$ $g(5)=$

$\lim_{x\to 2^-} g(x)=$ $\qquad$ $\lim_{x\to 5^-} g(x)=$

$\lim_{x\to 2^+} g(x)=$ $\qquad$ $\lim_{x\to 5^+} g(x)=$

$\lim_{x\to 2} g(x)=$ $\qquad$ $\lim_{x\to 5} g(x)=$

C. Average Velocity

DEFN: $$Velocity = \frac{Distance}{Time}$$

Example:

A ball is thrown up straight into the air with an initial velocity of 55 ft/sec, its height in feet t seconds is given by $y = 75t - 16t^2$.

a.) Find the average velocity for the period beginning when t=2 and lasting

(i) 0.1 seconds (i.e. the time period [2,2.1])

(ii) 0.01 seconds

(iii) 0.001 seconds

b.) Estimate the instantaneous velocity of the ball when t=2.