Sec 1.4 Calculating Limits

A. Limit Laws

Assume that f and g are functions and c is a constant.

1.) $\lim\limits_{x \to a} (f(x) + g(x)) = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$

2.) $\lim\limits_{x \to a} (f(x) - g(x)) = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$

3.) $\lim\limits_{x \to a} (c \cdot f(x)) = c \cdot \left(\lim\limits_{x \to a} f(x) \right)$

4.) $\lim\limits_{x \to a} (f(x) \cdot g(x)) = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$

5.) $\lim\limits_{x \to a} \left(\dfrac{f(x)}{g(x)} \right) = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$; $\lim\limits_{x \to a} g(x) \neq 0$

6.) $\lim\limits_{x \to a} (f(x))^n = \left(\lim\limits_{x \to a} f(x) \right)^n$

7.) $\lim\limits_{x \to a} c = c$

8.) $\lim\limits_{x \to a} x = a$

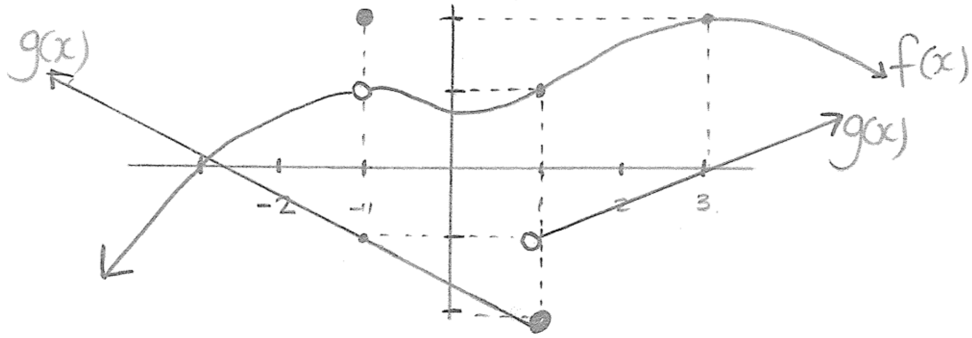
9.) $\lim\limits_{x \to a} x^n = a^n$

10.) $\lim\limits_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$

11.) $\lim\limits_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to a} f(x)}$

Example:

1.)



a.) $\lim_{x \to 1^+} 3g(x) =$

b.) $\lim_{x \to 1} f(x) =$

c.) $\lim_{x \to 1} \left(f(x) + g(x) \right) =$

d.) $\lim_{x \to 3} \left(f(x) \cdot g(x) \right) =$

2.) Given $\lim_{x \to a} h(x) = 1$, $\lim_{x \to a} f(x) = 10$, $\lim_{x \to a} g(x) = 0$.

a.) $\lim_{x \to a} \dfrac{h(x)}{f(x)}$

b.) $\lim_{x \to a} f(x)^{-1}$

c.) $\lim_{x \to a} \sqrt{f(x)}$

d.) $\lim_{x \to a} \dfrac{1}{f(x) - g(x)}$

e.) $\lim_{x \to a} \dfrac{g(x)}{h(x)}$

B. Calculating Limits

Direct Substitution Property: If f is a polynomial or rational function and $a \in$ Domain, then $\lim_{x \to a} f(x) = f(a)$

Examples:

1.) $\lim_{x \to 2} x^2 + 3x + 1 =$

2.) $\lim_{x \to 5} \frac{2x+2}{x-3} =$

3.) $\lim_{\theta \to \pi} \cos\theta =$

4.) $\lim_{x \to 4} \frac{x^2 - 16}{x-4} =$

5.) $\lim_{x \to 0} \frac{(x+2)^2 - 4}{2x} =$

6.) $\lim_{x \to 0} \frac{\sqrt{x+16} - 4}{x} =$

<u>Theorem</u> We say that a limit exists when the limit from the left equals the limit from the right.

$$\lim_{x \to a} f(x) = L \quad \Leftrightarrow \quad \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

Examples:

1.) $h(x) = \begin{cases} x^2 & if \;\; x \neq 2 \\ -1 & if \;\; x = 2 \end{cases}$ Find $\lim_{x \to 2} h(x)$

Theorem Squeeze Theorem: If $f(x) \le g(x) \le h(x)$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ when x is near a, then $\lim_{x \to a} g(x) = L$

Examples:

1.) Find $\lim_{x \to 0} x \cdot \cos\left(\frac{1}{x}\right)$

More Examples:

1.) $\lim_{x \to -8^-} \frac{5x+40}{|x+8|} =$

$\lim_{x \to -8^+} \frac{5x+40}{|x+8|} =$

$\lim_{x \to -8} \frac{5x+40}{|x+8|} =$

2.) Evaluate $\lim_{x \to 0} \frac{\tan(5x)}{\sin(6x)} =$