Sec 1.5 Continuity

A. Definition of Continuity

DEFN: A function f is continuous at a number a if:

(i) $f(a)$ exists

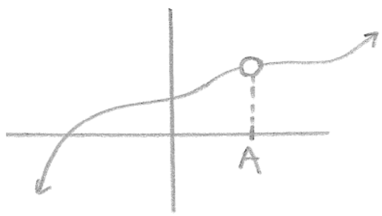
(ii) $\lim_{x\to a} f(x)$ exists

(iii) $\lim_{x\to a} f(x) = f(a)$

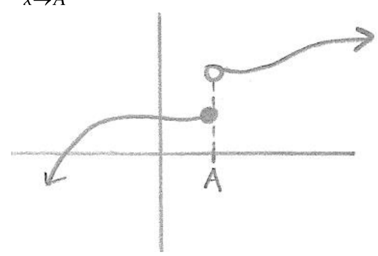
A function is defined as continuous only if it is continuous at every point in the domain of the function.

Examples: For each, determine whether the function is continuous (i.e. Is $\lim_{x\to A} f(x) = f(A)$?)

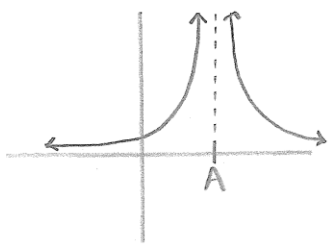
1.)



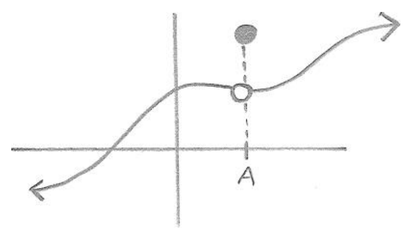
2.)



3.)



4.)



Examples: For each, determine whether the function is continuous. If not, where is the discontinuity?

1.) $f(x) = \dfrac{x^2 - x - 20}{x - 5}$

2.) $f(x) = \begin{cases} -6 - x & \text{if } x \le -3 \\ x & \text{if } -3 < x \le 3 \\ (x-1)^2 & \text{if } x > 3 \end{cases}$

3.) $h(x)=\begin{cases} x^2+1 & if \ x \neq 2 \\ 5 & if \ x=2 \end{cases}$

4.) $k(x)=\|x\|$ (Step function: i.e. $k(x)=\text{int}(x)$)

5.) For what value of the constant c is the function f continuous on $(-\infty,\infty)$ where

$$f(x)=\begin{cases} cx+7 & \text{if } x \in (-\infty,8] \\ cx^2-7 & \text{if } x \in (8,\infty) \end{cases}$$

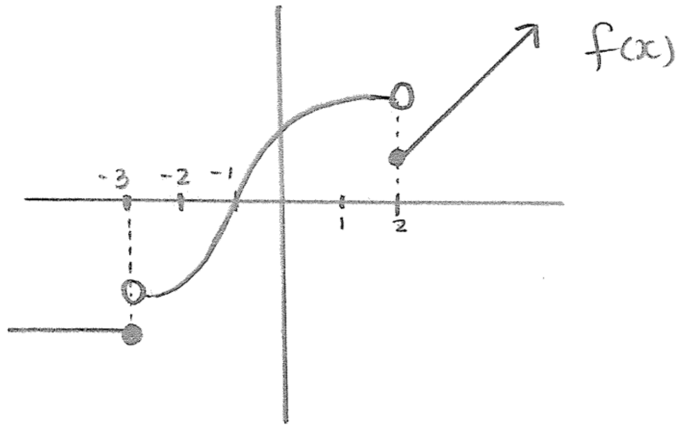
6.) Evaluate $\lim_{x \to 16} \frac{16-x}{4-\sqrt{x}} =$

DEFN: A function f is continuous from the RIGHT at a number a if: $\lim_{x \to a^+} f(x) = f(a)$

A function f is continuous from the LEFT at a number a if: $\lim_{x \to a^-} f(x) = f(a)$

Example:

1.)



Is f is continuous from the LEFT or RIGHT at

a.) $x = -3$

b.) $x = 2$

2.) Show that $f(x)$ has a jump discontinuity at $x = 9$ by calculating the limits from the left and right at $x = 9$.

$$f(x) = \begin{cases} x^2 + 5x + 5, & \text{if } x < 9 \\ 14, & \text{if } x = 9 \\ -4x + 4, & \text{if } x > 9 \end{cases}$$

<u>Theorem</u> If f and g are functions that are continuous at a number a, and c is a constant, then the following are also continuous at a:

(i) $(f + g)$

(ii) $(f - g)$

(iii) $(f \cdot g)$

(iv) $\left(\frac{f}{g}\right)$ if $g(a) \neq 0$

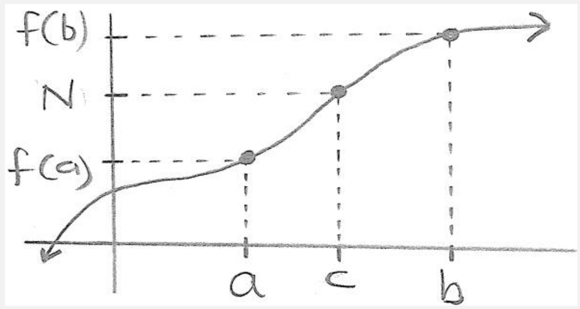
(v) $c \cdot f$ or $c \cdot g$

<u>Theorem</u> A **polynomial** function is continuous everywhere

A **rational** function is continuous everywhere it is defined

Theorem   Intermediate Value Theorem

If f is a function that is continuous on a closed interval $[a,b]$ where $f(a) \neq f(b)$ and N is a number such that $f(a) < N < f(b)$. Then there exist a number c such that $a < c < b$ and $f(c) = N$.



Examples:

1.) Show that $f(x) = x^2 - x - 2$ has a root on the interval $[1,3]$

2.) Let f be a continuous function such that $f(1) < 0 < f(9)$. Then the Intermediate Value Theorem implies that $f(x) = 0$ on the interval (A, B). Give the values of A and B.