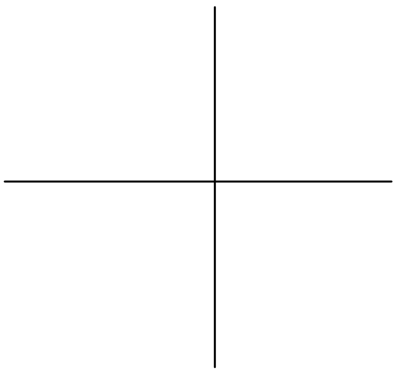
Sec 1.6 Limits Involving Infinity

A. Infinity vs. DNE

Recall from section 1.3 that $\lim_{x \to 0} \frac{1}{x^2}$ DNE since the function value kept increasing. Now we will be more descriptive; any value that keeps increasing is said to approach infinity (∞), and any value that keeps decreasing is said to approach negative infinity ($-\infty$).

Examples:

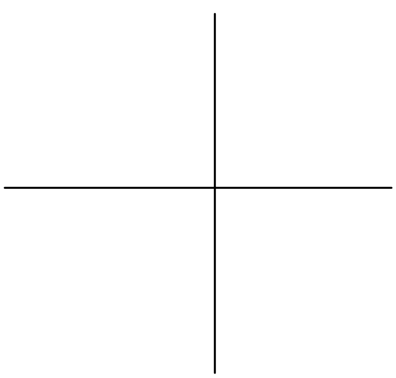
1.) Evaluate $\lim_{x \to 0} \frac{1}{x^2}$ using the graph and table method.



$\lim_{x \to 0^-} \frac{1}{x^2}$	
x	y
-0.1	
-0.01	
-0.001	

$\lim_{x \to 0^+} \frac{1}{x^2}$	
x	y
0.1	
0.01	
0.001	

2.) Evaluate $\lim_{x \to 0} \frac{1}{x}$ using the graph and table method.



$\lim_{x \to 0^-} \frac{1}{x}$	
x	y
-0.1	
-0.01	
-0.001	

$\lim_{x \to 0^+} \frac{1}{x}$	
x	y
0.1	
0.01	
0.001	

B. A Quick Review of Asymptotes

An asymptote is an imaginary line that the graph of a function approaches as the function approaches a restricted number in the domain or as it approaches infinity.

Locating Vertical Asymptotes

If $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function, $p(x)$ and $q(x)$ have no common factors and n is a zero of $q(x)$, then the line $x = n$ is a vertical asymptote of the graph of $f(x)$.

Locating Horizontal Asymptotes.

Let $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0}$

i. If n < m, then $y = 0$ is the horizontal asymptote

ii. If n = m, then the line $y = \dfrac{a_n}{b_m}$ is the horizontal asymptote

iii. If n > m, there is NO horizontal asymptote. (But there will be a slant/oblique asymptote.)

Examples: For the following rational functions, find the vertical and horizontal asymptotes if any:

1.) $f(x) = \dfrac{16x^2}{4x^2 + 1}$

2.) $g(x) = \dfrac{x + 8}{x^2 - 64}$

3.) $h(x) = \dfrac{x^3 + 7}{5x - 2}$

4.) $k(x) = \dfrac{x^2 - 2x}{2 - 3x + x^2}$

C. Vertical Asymptotes

Vertical asymptotes occur when $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$

The asymptote will be the line $x = a$.

Example: Evaluate the limit, find the asymptote and graph the function

1.) $\lim_{x \to 2} \frac{x+1}{3x-6}$

2.) $\lim_{x \to -4^+} \frac{x+6}{x+4}$

3.) $\lim_{x \to -4^-} \frac{x+6}{x+4}$

D. Limits as Infinity

A limit as the domain approaches infinity: $\lim_{x \to \infty} f(x)$

Finding Limits as Infinity of Rational Functions

i. Determine the degree of the denominator. (Let's say degree = P)

ii. Multiply both the numerator and denominator by $\frac{1}{x^P}$.

iii. Distribute/clean up algebra and continue evaluating the limit.

Example: Evaluate the limit.

1.) $\lim_{x\to\infty} \frac{6x^2+2x+7}{8x+2x^2}$

2.) $\lim_{x\to\infty} \frac{x^3+4x-2}{6-2x^2}$

3.) $\lim_{x\to\infty} \frac{2x}{2x^2+x-1}$

<u>Conclusion</u>: *For positive integers* M *and* N *such that* $M > N$

1. Degree of the Numerator = Degree of the Denominator
$$\lim_{x\to\infty} \frac{Polynomail\ of\ Degree\ M}{Polynomail\ of\ Degree\ M} = Ratio\ of\ Leading\ Coeficient\, s$$
2. Degree of the Numerator > Degree of the Denominator
$$\lim_{x\to\infty} \frac{Polynomail\ of\ Degree\ M}{Polynomail\ of\ Degree\ N} = \pm\infty$$
3. Degree of the Numerator < Degree of the Denominator
$$\lim_{x\to\infty} \frac{Polynomail\ of\ Degree\ N}{Polynomail\ of\ Degree\ M} = 0$$

More Example: Evaluate the limit.

1.) $\lim_{x \to \infty} \frac{3x-10}{\sqrt{16x^2+5}}$

2.) $\lim_{x \to -\infty} \frac{\sqrt{9x^2+x+1}}{2x+1}$

3.) Find the horizontal asymptotes for the curve $y = \frac{12x}{\left(x^4+1\right)^{\frac{1}{4}}}$

4.) Find the vertical asymptotes for the curve $y = \frac{4x^3}{x+2}$

5.) $\lim_{x \to \frac{\pi}{2}^-} \tan x$

6.) $\lim_{x \to \frac{\pi}{2}^+} \tan x$

7.) $\lim_{x \to \infty} \sqrt{x^2 + 7x + 1} - x$