Sec 2.1 Derivatives and Rates of Change

A. Slope of Secant Functions

Recall: Slope = m = $\frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. From this we are able to derive:

> Slope of the Secant Line to a Function: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Examples:

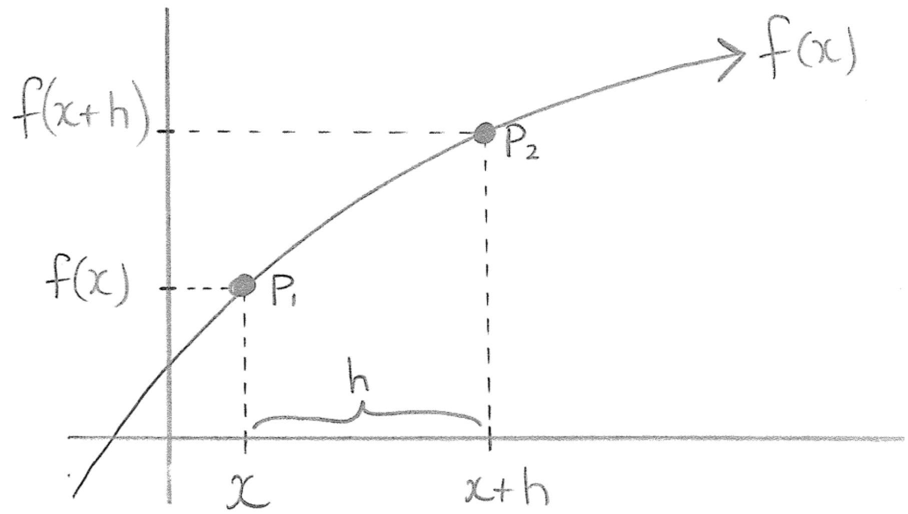
1. a.) Find the **slope** of the secant line to the function $f(x) = \sqrt{x}$ between $x = 1$ and $x = 9$

b.) Find the **equation** of the secant line to the function $f(x) = \sqrt{x}$ between $x = 1$ and $x = 9$

2.) Estimate the equation of the **tangent** line to the function $y = x^2$ at the point $(1, 1)$ by calculating the equation of the **secant** line between $x = 1$ and $x = 1.1$, between $x = 1$ and $x = 1.01$ and between $x = 1$ and $x = 1.001$.

A generalization of the previous example gives the definition: Slope = $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

The slope of the tangent line can alternatively be calculated in the following way:



In order to get the two points P_1 and P_2 as close together as possible, we need for the space h→0.
So, the slope between P_1 and P_2 is:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - (x)} = \frac{f(x+h) - f(x)}{h}$$

but as the space h→0, we have

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

B. Definition of Derivative

The definition of a derivative (a.k.a. the slope of the tangent function) is given as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

*Note: In this section we will use the DEFINITION OF THE DERIVATE to calculate all derivatives. (This means we will be doing it the long way!)

Examples:

1.) Find the equation of the tangent line to the function $f(x) = x^2 - 3x + 1$ where $x = 5$.

2.) A person standing on top of a 200 ft tall building throws a ball into the air with a velocity of 96 ft/sec. The function $s(t) = -16t^2 + 96t + 200$ gives the ball's height above ground, t seconds after it was thrown. Find the instantaneous velocity of the ball at t = 2 seconds

3.) The position of a particle is given by the values of the table.

t(seconds)	0	1	2	3	4	5
s(feet)	0	14	47	51	86	103

Find the average velocity for the time period beginning when t=2 and lasting

1. 3 s (i.e. for the time interval [2,5])
2. 2 s
3. 1 s

4.) (a) The equation of the tangent line to the graph of $y = g(x)$ at $x = 3$ if $g(3) = -3$ and $g'(3) = 7$ is $y =$

(b) If the tangent line to $y = f(x)$ at (2, 10) passes through the point (0,4), then $f(2) =$
and $f'(2) =$

5.) $\lim_{h \to 0} \frac{\sqrt{81+h}-9}{h}$ represents the derivative of the function $f(x) = \sqrt{x}$ at the number $a =$

6.) $\lim_{x \to 6} \frac{2^x - 64}{x-6}$ represents the derivative of the function $f(x) =$_____ at the number $a =$