Sec 2.2 The Derivative of a Function

A. Definition of the Derivative

For a function $f(x)$ the derivative is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

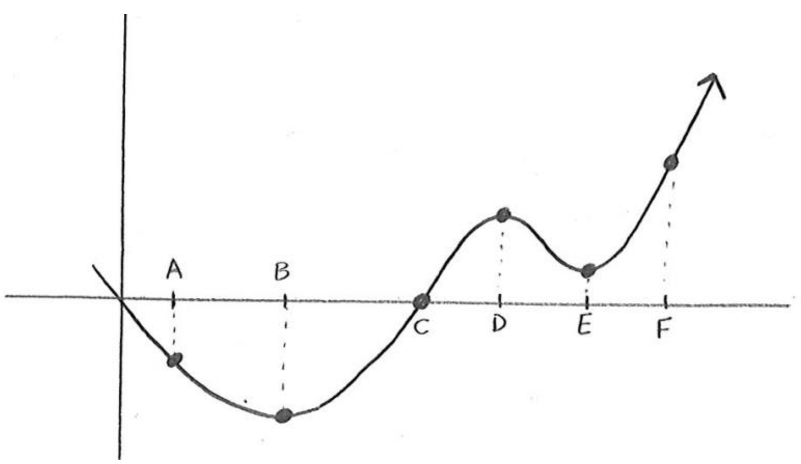
Examples: Using the definition of the derivative, find the derivative of the following functions:

1.) $f(x) = x^2 + 3x - 2$

2.) $g(x) = \sqrt{x} + 4$

3.) $k(x)=\frac{1}{2x+1}$

4.) Consider the graph for the function $f(x)$



Estimate the following:

a.) $f'(A)=$

b.) $f'(B)=$

c.) $f'(C)=$

d.) $f'(D)=$

e.) $f'(E)=$

f.) $f'(F)=$

B. Differentiability

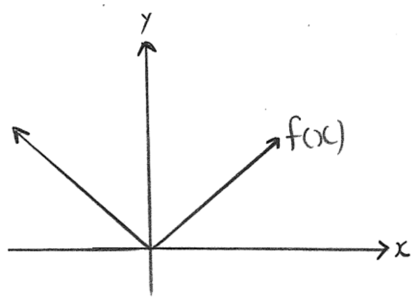
DEFN: We say that

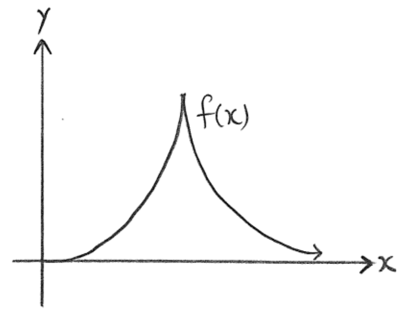
- $f(x)$ is differentiable at a if $f'(a)$ exists
- $f(x)$ is differentiable on $(a\ ,b)$ if it is differentiable on every point in $(a\ ,b)$

Theorem: If f is differentiable at a , then f is continuous at a

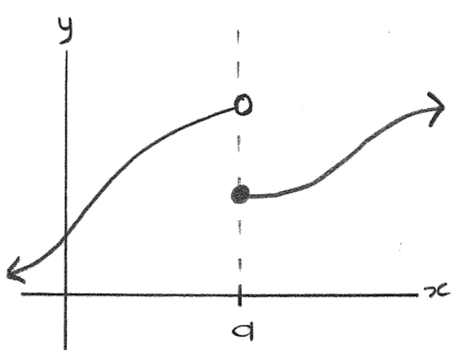
Non differentiable functions:

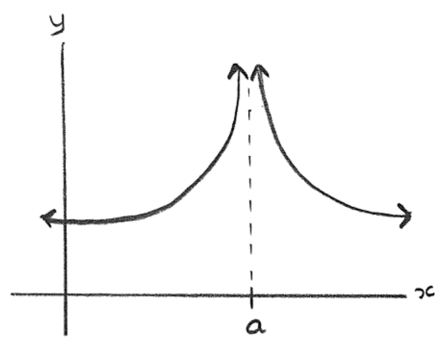
i. Any function with a "corner" or cusp



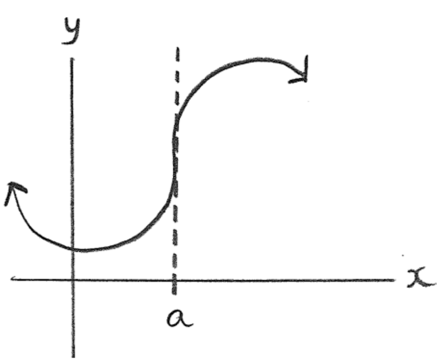


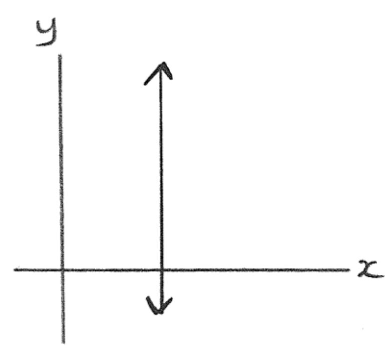
ii. Any function with a discontinuity





iii. Any function with a vertical tangent





C. Notation

Function	Derivative
$y =$	$y' =$ $\frac{dy}{dx} =$ (Leibniz notation)
$f(x)$	$f'(x)$
$F(x)$	$f(x)$

D. Higher Derivatives

(i.e. Finding the "derivative of a derivative" or taking the derivative multiple times.)

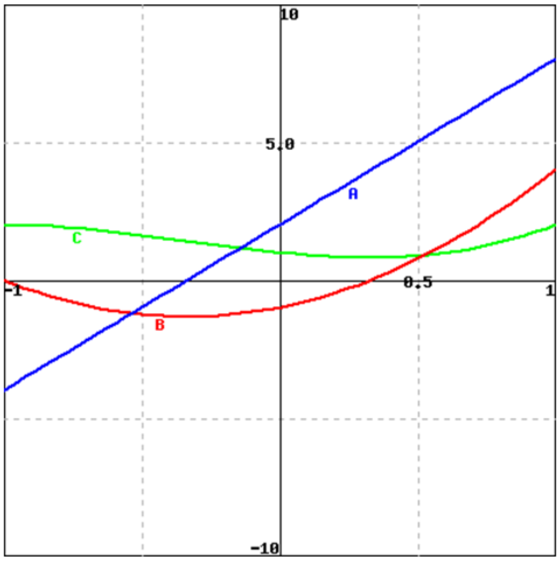
$$\frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d^2y}{dx^2} = y'' = f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

Example:

1.) For the function $f(x) = x^2 + 3x - 2$, find the second derivative. (Recall from previous that $f'(x) = 2x + 3$)

More Examples:

1.)



Identify the graphs A (blue), B(red) and C (green) as the graphs of a function and its derivatives.

a.) The graph of the function is:

b.) The graph of the function's first derivative is:

c.) The graph of the function's second derivative is:

2.) Evaluate the limit $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$

3.) Evaluate the limit $\lim_{x \to 0} \frac{\sqrt{x^2 + 36} - 6}{x^2}$