A. Properties and Formulas (The short way – Yeah!)

1. Basic Functions

Function	Derivative
$f(x)=c$ (Constant)	$f'(x)=0$
$f(x)=x$	$f'(x)=1$
$f(x)=cx$	$f'(x)=c$
$f(x)=x^n$	$f'(x)=nx^{n-1}$
$f(x)=cx^n$	$f'(x)=cnx^{n-1}$
$f(x)=c\cdot g(x)$	$f'(x)=c\cdot g'(x)$
$f(x)=g(x)+h(x)$	$f'(x)=g'(x)+h'(x)$
$f(x)=g(x)-h(x)$	$f'(x)=g'(x)-h'(x)$

Note: For the function $f(x)=g(x)\cdot h(x)$ we CAN NOT say that $f'(x)=g'(x)\cdot h'(x)$

For the function $f(x)=\dfrac{g(x)}{h(x)}$ we CAN NOT say that $f'(x)=\dfrac{g'(x)}{h'(x)}$

2. Trigonometric Functions

Function	Derivative
$f(x)=\sin(x)$	$f'(x)=\cos(x)$
$f(x)=\cos(x)$	$f'(x)=-\sin(x)$
$f(x)=\tan(x)$	$f'(x)=\sec^2(x)$
$f(x)=\csc(x)$	$f'(x)=-\csc(x)\cot(x)$
$f(x)=\sec(x)$	$f'(x)=\sec(x)\tan(x)$
$f(x)=\cot(x)$	$f'(x)=-\csc^2(x)$

Examples: Find and LABEL the derivatives of each of the following functions.

1.) $f(x) = 8$

2.) $g(x) = 8x$

3.) $k(x) = x^2$

4.) $h(x) = 5x^3$

5.) $v(r) = \frac{3}{4}\pi r^3$

6.) $q(y) = y$

7.) $m(x) = \sqrt{x}$

8.) $a(x) = 4x^{\frac{3}{2}}$

9.) $v(t) = \frac{1}{t}$

10.) $d(x) = \frac{1}{x^3}$

11.) $p(x) = \frac{x^3 + 4x^2}{x}$

12.) $d(x) = (x+3)^2$

More Examples

13.) $f(x)=x^3+4x^2-2x+4$ Find $f''(1)$

14.) $g(t)=\dfrac{4t^2+t+5}{\sqrt{t}}$ Find $g''(t)$

15.) $h(x)=x^5-2x^4+3x^3-x-6$ Find the first 5 derivatives of the function.

B. Normal and Tangent Lines to a Function

Normal Line: A line that is perpendicular to the tangent line.

Examples:

1.) Find the tangent and the normal lines to the function $f(x) = 4\cos(x)$ at $x = \frac{\pi}{3}$

2.) Find the horizontal tangent lines (lines with slope = 0) to the function $f(x) = 2x^3 + 3x^2 - 120x + 23$

C. Applications to Position, Velocity and Acceleration

If the motion/position function of a particle is known, we can find the velocity and acceleration functions in the following way.

- If the **position** of a particle is given by $f(x)$, then the **velocity** of the particle is given by $f'(x)$
- If the **velocity** of a particle is given by $g(x)$, then the **acceleration** of the particle is given by $g'(x)$
  (We can also say that If the position of a particle is given by $f(x)$, then the acceleration of the particle is given by $f''(x)$, the second derivative of the motion function.)

Alternative notation:

- **Position** of a particle $s(t)$
- **Velocity** of a particle $v(t)$
- **Acceleration** of a particle $a(t)$

Then $v(t) = s'(t)$ And $a(t) = v'(t) = s''(t)$

Example:

1.) A particle's **position** is described by the function $s(t) = 3t^3 - 144t$. (t is measures in seconds and $s(t)$ in feet.)

a. Find the velocity function.

b. Find the acceleration function.

c. Find the acceleration after 9 seconds.

d. Find the acceleration when the velocity is 0.

2.) A particle's **position** is described by the function $f(t)=t^3-9t^2+15t+10$. (t is measures in seconds and $s(t)$ in feet.)

a. Find the velocity function.

b. What is the velocity after 3 seconds?

c. When is the particle at rest?

d. When is the particle moving in a positive direction?

e. When is the particle slowing down?

f. Find the total distance traveled during the first 8 seconds.

3.) The area of a disc with radius r is $A(r)=\pi r^2$. Find the rate of change of the area of the disc with respect to its radius when $r=5$.

More Examples:

1.) If $f(x)=(\frac{3}{4}x)^9$, then $f'(x)=$

2.) If $f(x)=\sqrt{x}(3x+0)$, then $f'(x)=$

3 a.) If $f(x)=12\pi^2$, then $f'(x)=$

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b.) If $f(x)=12x^2$, then $f'(x)=$

c.) If $f(x)=12\pi x^2$, then $f'(x)=$

4 If a ball is thrown vertically upward from the roof of 64 foot building with a velocity of 32 ft/sec, its height after t seconds is $s(t) = 64 + 32t - 16t^2$.

a.) What is the maximum height the ball reaches? .

b.) What is the velocity of the ball when it hits the ground (height 0)?

5 Evaluate the following limits:

a.) $\lim_{x \to 2} \frac{x^5 - 32}{x - 2} =$

b.) $\lim_{x \to 1} \frac{x^{615} - 1}{x - 1} =$