A. Product Rule

$$f(x)=g(x)\cdot h(x) \text{ then } f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)$$

Alternative Notation: $\frac{d(g\cdot h)}{dx}=\left(\frac{dg}{dx}\right)\cdot(h)+(g)\cdot\left(\frac{dg}{dx}\right)$

In Plain English: The derivative of the product of two functions (which we will call the "first" function and the "second" function) is equal to the **derivative of the first, times the second, plus the first, times derivative of the second**.

Examples

1.) $f(x)=(2x^3+8x)\cdot(5x^4+17)$

We see that $f(x)$ consists of the product of two smaller functions, in this case $\underline{(2x^3+8x)}$ "the first" and $\underline{(5x^4+17)}$ "the second". So, the derivative then is:

$$f'(x)=\underbrace{(6x^2+8)}_{\textbf{Derivative of the first}}\ \text{x}\ \underbrace{(5x^4+17)}_{\textbf{The second}}+\underbrace{(2x^3+8x)}_{\textbf{The first}}\ \text{x}\ \underbrace{(20x^3)}_{\textbf{Derivative of the second}}$$

Note: You should leave the answer in this form unless we are asked to "clean up"
Again, do not forget to label your derivative

2.) $g(x)=x\cdot\sin(x)$

We see that $g(x)$ consists of the product of two smaller functions, in this case $\underline{x}$ "the first" and $\underline{\sin(x)}$ "the second".

So, the derivative then is: $g'(x)=(1)\cdot\sin(x)+x\cdot\cos(x)=\sin(x)+x\cdot\cos(x)$

More Examples: Find and LABEL the derivatives of each of the following functions.

1.) $f(x)=(x^4+\sqrt{x})\cdot(5x-1)$

2.) $f(x)=x^2\cos(x)$

3.) $f(x)=\sin(x)\cos(x)$

B. Quotient Rule

$$f(x)=\frac{g(x)}{h(x)} \text{ then } f'(x)=\frac{g'(x)\cdot h(x)-g(x)\cdot h'(x)}{[h(x)]^2}$$

Book Notation: $$\frac{d\left(\frac{g}{h}\right)}{dx}=\frac{\left(\frac{dg}{dx}\right)\cdot(h)-(g)\cdot\left(\frac{dg}{dx}\right)}{h^2}$$

In Plain English: The derivative of the quotient of two functions (which we will call the "top" function and the "bottom" function) is equal to the **derivative of the top, times the bottom, minus the top, times derivative of the bottom, all over the bottom squared.**

Examples

1.) $f(x)=\frac{(2x^3+8x)}{(5x^4+17)}$

We see that $f(x)$ consists of the quotient of two smaller functions, in this case $\underline{(2x^3+8x)}$ "the top" and $\underline{(5x^4+17)}$ "the bottom". So, the derivative then is:

$$f'(x)=\frac{\overbrace{(6x^2+8)}^{\text{Derivative of the top}} \times \overbrace{(5x^4+17)}^{\text{The bottom}} - \overbrace{(2x^3+8x)}^{\text{The top}} \times \overbrace{(20x^3)}^{\text{Derivative of the bottom}}}{\underbrace{(5x^4+17)^2}_{\text{The bottom squared}}}$$

Note: You should leave the answer in this form unless we are asked to "clean up"
Again, do not forget to label your derivative

2.) $g(x)=\frac{x}{\sin(x)}$

We see that $g(x)$ consists of the product of two smaller functions, in this case $\underline{x}$ "the top" and $\underline{\sin(x)}$ "the bottom".

So, the derivative then is: $g'(x)=\frac{(1)\cdot\sin(x)-x\cdot\cos(x)}{[\sin(x)]^2} = \frac{\sin(x)-x\cdot\cos(x)}{\sin^2(x)}$

More Examples: Find and LABEL the derivatives of each of the following functions.

1.) $f(x)=\frac{(x^4-8x)}{(2x-1)}$

2.) $g(x)=\frac{\sin x}{\cos x}$

3.) $l(x)=\frac{\sin x}{\sqrt{x}}\cdot(x^2+2x)$

4.) Suppose $f(\pi/6)=7$ and $f'(\pi/6)=-5$, and let $g(x)=f(x)\cos x$ and $h(x)=\frac{\sin x}{f(x)}$

$g'(\pi/6)=$

$h'(\pi/6)=$

5.)

x	1	0	7	-2	-1
$f(x)$	-5	-1	-407	7	1
$g(x)$	-3	-2	-9	0	-1
$f'(x)$	-7	-2	-163	-10	-3
$g'(x)$	-1	-1	-1	-1	-1

a. $(fg)'(-1)$

b. $f(-1)/(g(-1)+5)$

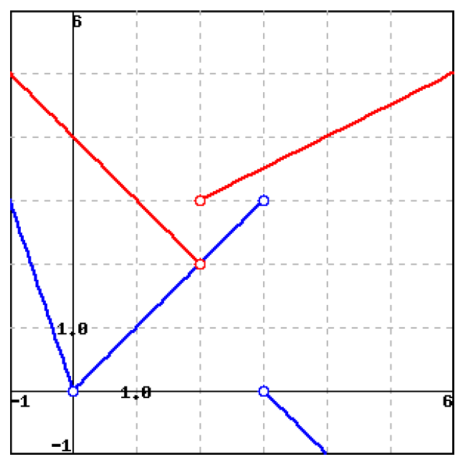
c. $(f+g)'(-1)$

d. $(f-g)'(-1)$

e. $(fg)'(7)$

f. $\left(\frac{g}{f}\right)'(0)$

6.) The graphs of the function f (given in blue) and g (given in red) are plotted above. Suppose that $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$. Find each of the following:



f is the bottom function
g is the top function

$u'(1) =$

$v'(1) =$

7.) Given that

$$f(x) = x^{12}h(x)$$
$$h(-1) = 3$$
$$h'(-1) = 6$$

Calculate $f'(-1)$