Sec 2.5 Chain Rule

A. The Chain Rule

$$[h(g(x))]' = h'(g(x)) \cdot g'(x)$$

Alternative Notation:

$f(x) = (h \circ g)(x) = h(g(x))$ then $f'(x) = [h'(g(x))] \cdot [g'(x)]$

$$[h(g(x))]' = \frac{d(h(g))}{dx} = \left(\frac{dh}{dg}\right) \cdot \left(\frac{dg}{dx}\right)$$

In Plain English: First, identify which function is on the "outside" and which is on the "inside". (For the composition $(f \circ g)(x) = f(g(x))$ we say that f is on the "outside" and g is on the "inside".) The derivative of this composition is equal to **the derivative of the outside (leave the inside alone) times the derivative of the inside**.

Examples

1.) $f(x) = \sin(5x^5)$

First, let us identify which is the "outside" and which is on the "inside": Here $\underline{\sin(\ldots)}$ is the "outside" (i.e. "sin of something") and $\underline{5x^5}$ is the "inside".

Derivative of the "outside" is $\cos(\ldots)$, and if we the leave the inside alone, this will be $\cos(5x^5)$

Derivative of the "inside" is $25x^4$

$$f'(x) = \underbrace{\left(\cos(5x^5)\right)}_{\text{Derivative of the outside (leave the inside alone)}} \times \underbrace{\left(25x^4\right)}_{\text{Derivative of the inside}}$$

Note: This style of answer should only be "cleaned up" if you are given specific instructions to so (or if you have to compare it to a list of multiple choice answers)!
Again, do not forget to label your derivative

More Examples

2.) $g(x)=\sin^2(x)=[\sin(x)]^2$

We see that $g(x)$ consists of $\underline{[\ldots]^2}$ as the "outside" and $\underline{\sin(x)}$ as the "inside".

So, the derivative of the "outside" is $2[\ldots]$ and derivative of the "inside" is $\cos(x)$

$$\Rightarrow \quad g'(x)=2[\sin(x)]\cdot[\cos(x)]$$

Examples: Find and LABEL the derivatives of each of the following functions.

1.) $f(x)=\tan(6x^3)$

2.) $k(x)=\tan(\sin x)$

3.) $l(x)=\sqrt{x^2+1}$

4.) $f(x)=(x^2+2x)^8$

5.) $r(s)=\dfrac{1}{\sqrt[3]{s^4+s}}$

B. Combinations of Product, Quotient and Chain Rule

In many problems we need to use a combination of the Product, Quotient and Chain Rule to find a derivative. Here we will work through lots of examples.

Examples: Find and LABEL the derivatives of each of the following functions. Do not clean up unless otherwise indicated.

1.) $g(x) = \sin(x^2) \cdot (8x^3 - 1)$

2.) $f(x) = \sin^3(x^2) \cdot$

3.) $g(x) = (x - \cos^2(x))^4$

4.) $r(\theta) = \dfrac{\cos(2\theta - 4\pi)}{\theta^2}$

5.) $a(x) = x^2 \cdot \sqrt{2x^3 + 1}$

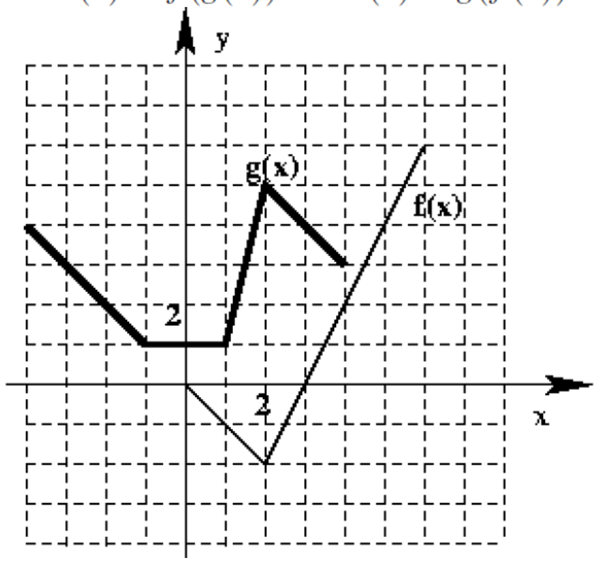
6.) $f(x) = \dfrac{5x^3}{(x^2 + 5x)^3}$

7.) If $f(x) = \cos(a^8 + x^8)$, then $f'(x) =$

8.) If $f(x) = \cos(x \sin x)$, find $f'(x) =$

9.) Let $F(x) = f(g(x))$, where $f(-7) = 2$, $f'(-7) = 2$, $f'(3) = 15$, $g(3) = -7$, and $g'(3) = -8$, find $F'(3)$

10.) If f and g are the functions whose graphs are shown below, let $u(x) = f(g(x))$ and $v(x) = g(f(x))$.



Find $u'(3)$

and $v'(3)$

11.) Let $F(x) = f(x^6)$ and $G(x) = (f(x))^6$. You also know that $a^5 = 11,\ f(a) = 2,$ $f'(a) = 8,\ f'(a^6) = 15.$

Find $F'(a)$

and $G'(a)$