Before getting started with Related Rates, let us re-visit the following items first: Notation, Implicit Differentiation and Geometric Formulas

A. Notation

$$f'(x) \Leftrightarrow y' \Leftrightarrow \frac{dy}{dx}$$

Although all of the above notations are equivalent, we will use Leibniz's notation $\left(\frac{dy}{dx}\right)$ in this section, because it is more descriptive than the other forms. Leibniz's notation tells us specifically what we are taking a derivative of (in this case the function y) and what we are taking the derivative with respect to (w.r.t.) – i.e. what is the variable in the function (in this case x.)

B. Implicit Differentiation

Again, we will have to pay close attention to notation here. In equations with multiple variables, we will be asked to find derivatives of specific parts of the equations with respect to specific variables (that may or may not be part of the equation!).

For Example: Consider the equation for the circle: $r^2 = x^2 + y^2$

- We would like to find $\frac{dr}{dt}$. This means, we are trying to find the derivative of **r** with respect to **t**. I.e. take a derivative of each term with respect to **t**. (If a term is/contains a **t**, just take a derivative as usual. If a term contains a variable other than **t**, follow the usual rules for implicit differentiation.)

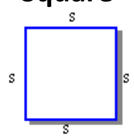
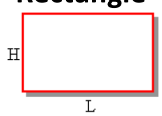
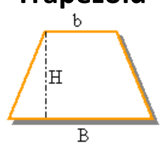
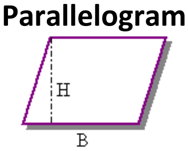
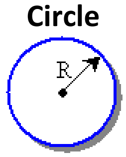
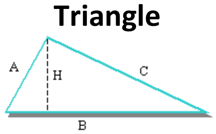
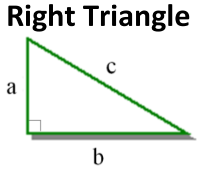
  So, $(2r)\frac{dr}{dt} = (2x)\frac{dx}{dt} + (2y)\frac{dy}{dt} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{(2x)\frac{dx}{dt} + (2y)\frac{dy}{dt}}{2r}$

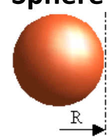
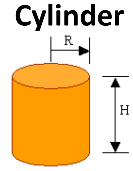
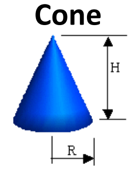
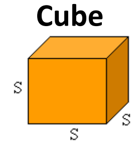
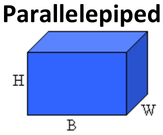
- We would like to find $\frac{dx}{dt}$.

  So, $(2r)\frac{dr}{dt} = (2x)\frac{dx}{dt} + (2y)\frac{dy}{dt} \quad \Rightarrow \quad \frac{dx}{dt} = \frac{(2r)\frac{dr}{dt} - (2y)\frac{dy}{dt}}{2x}$

C. Geometric Formulas

* You are responsible for knowing these formulas for all tests and the final exam*

2-Dimentional Shapes	
Shape	**Perimeter/Circumference and Area**
Square 	$P = 4S$ $A = S^2$
Rectangle 	$P = 2H + 2L$
Trapezoid 	$A = \dfrac{H(B+b)}{2}$
Parallelogram 	$A = BH$
Circle 	$C = 2\pi R$ $A = \pi R^2$
Triangle 	$A = \dfrac{BH}{2}$
Right Triangle 	$c^2 = a^2 + b^2$

3-Dimentional Shapes	
Shape	
Sphere 	$SA = 4\pi R^2$ $V = \dfrac{4}{3}\pi R^3$
Cylinder 	$SA = 2\pi RH + 2\pi R^2$ $V = \pi R^2 H$
Cone 	$V = \dfrac{\pi R^2 H}{3}$
Cube 	$SA = 6S^2$ $V = S^3$
Rectangular Parallelepiped 	$SA = 2HB + 2BW + 2HW$ $V = BHW$

D. Related Rate Problems

1. If $z^2 = x^2 + y^2$, $\frac{dx}{dt} = 9$, and $\frac{dy}{dt} = 5$, Find $\frac{dz}{dt}$ when $x = 2$ and $y = 5$

2. Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1.5 m/s, how fast is the area of the spill increasing when the radius is 19 m?

Hint: The word **rate = derivative**. Pay attention to units to find out which rate is given asked for.

(Example, “rate of 1.5 m/s” – meters per second = unit of **length** per unit of **time =** $\frac{d(length)}{d(time)} = \frac{d(r)}{d(t)}$)

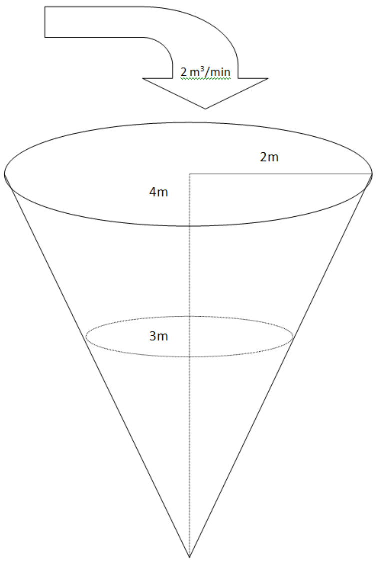
3. A fireman is on top of a 75 foot ladder that is leaning against a burning building. If someone has tided Sparky (the fire dog) to the bottom of the ladder and Sparky takes off after a cat at a rate of 6 ft/sec, then what is the rate of change of the fireman on top of the ladder when the ladder is 5 feet off the ground?

4. A street light is mounted at the top of a 11 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 8 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 50 ft from the base of the pole?

5. If a snowball melts so that its surface area decreases at a rate of $.01\frac{cm^2}{\text{min}}$, find the rate at which the diameter decreases when the diameter is 8cm.

6. At noon, ship A is 30 miles due west of ship B. Ship A is sailing west at 25 mph and ship B is sailing north at 18 mph. How fast (in mph) is the distance between the ships changing at 5 PM?

7. Water pours into in inverted cone at a rate of 2 m^3/min. It the cone has a radius of 2 m and a height of 4 m, find the rate at which the water level is rising when the water is 3 m deep.



8.) A boat is pulled into a dock by means of a rope attached to a pulley on the dock. The rope is attached to the front of the boat, which is 7 feet below the level of the pulley. If the rope is pulled through the pulley at a rate of 20 ft/min, at what rate will the boat be approaching the dock when 120 ft of rope is out?

* Examples on this topic available at Justmathtutoring.com > Free Calculus Videos > Related Rates - … *