Sec 2.8 Linear Approximation and Differentials

A. Differentials

Slope = $\dfrac{\Delta y}{\Delta x}$

Slope of the tangent line = $\dfrac{dy}{dx}$

$\Longrightarrow$ $\dfrac{\Delta y}{\Delta x} \approx \dfrac{dy}{dx}$

Also, $\dfrac{dy}{dx} = f'(x)$ $\Longrightarrow$ $dy = [f'(x)] \cdot dx$

Example: Find the differential of y given that:

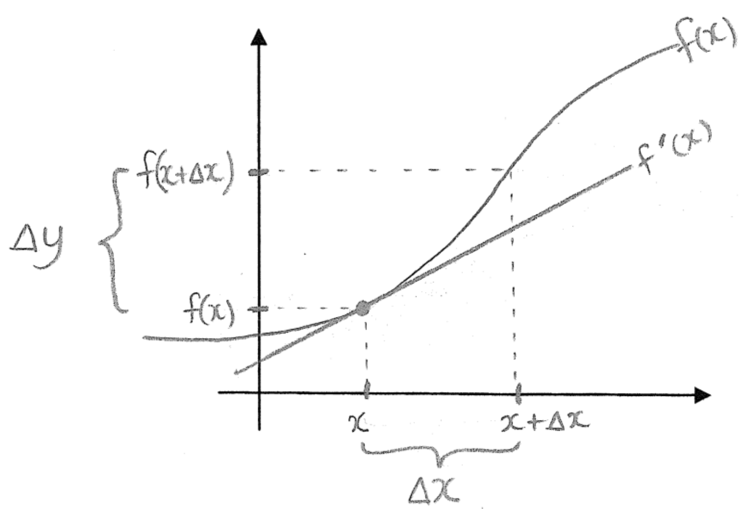
1.) $y = 5x^3$

2.) $y = \dfrac{3x^2 + 4x - 5}{5\sin(x) + 2x}$

Note:

$\Delta x \approx dx$ and $\Delta y \approx dy$

$$dx \approx \Delta x = (x + \Delta x) - (x)$$
$$dy \approx \Delta y = f(x + \Delta x) - f(x)$$



Example: $y = x^3$

1a.) Calculate Δy for $x = 2$ to $x = 2.01$

b.) Calculate dy for $x = 2$ to $x = 2.01$

2.) $V = \frac{4}{3}\pi r^3$ (volume of a sphere)

a.) Use differentials to approximate the change in volume when going from $r = 3\,ft$ to $r = 2.8\,ft$

(Find dV : Start by finding $\frac{dV}{dr}$)

b.) Find the actual change in volume when going from $r = 3\,ft$ to $r = 2.8\,ft$

(Find ΔV)

B. Linearization

Definition: The linearization of a function $f(x)$ at a fixed point a is given by the formula

$$L(x) = f(a) + f'(a)(x-a)$$

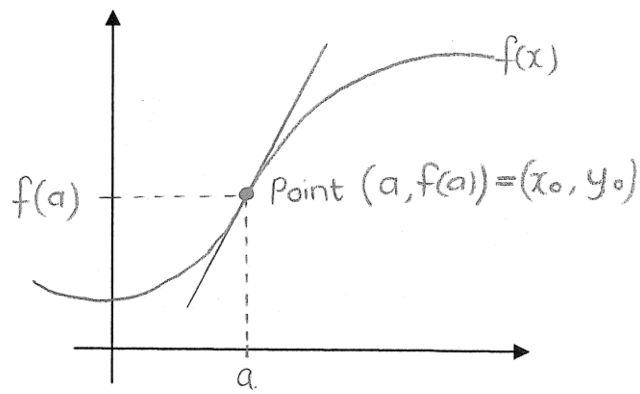
Slope of the tangent line = $m_T = f'(a)$

Point Slope: $(y - y_o) = m_T(x - x_0)$

$$(y - f(a)) = f'(a)(x-a)$$

$$y = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$



Examples:

1.) Find the linear approximation of $f(x) = \cos(5x)$ at $a = \frac{\pi}{2}$

2.) Use linearization techniques to approximate $\sqrt{16.1}$

More Examples:

3.) Find the linear approximation of $f(x) = \sqrt{4-x}$ at $a = 0$ and use it to approximate $\sqrt{3.9}$ and $\sqrt{4.1}$.

4.) Use a linear approximation to approximate 2.001^6 as follows:
The linearization $L(x)$ to $f(x) = x^6$ at $a = 2$ can be written in the form $L(x) = mx + b$
Using this, the approximation for 2.001^6 is

5.) The edge of a cube was found to be 60 cm with a possible error of 0.5 cm. Use differentials to estimate:

(a) the maximum possible error in the volume of the cube

(b) the relative error in the volume of the cube

(c) the percentage error in the volume of the cube

6.) A 13 foot ladder is leaning against a wall. If the top slips down the wall at a rate of 4 ft/s, how fast will the foot of the ladder be moving away from the wall when the top is 8 feet above the ground?