Sec 3.7 Indeterminate forms and L'Hospital's Rule

A. Indeterminate forms

If we have a limit of the form $\lim_{x\to a}\frac{f(x)}{g(x)}$ where both $f(x)\to 0$ and $g(x)\to 0$, then we have the in determinant form of type $\frac{0}{0}$

If we have a limit of the form $\lim_{x\to a}\frac{f(x)}{g(x)}$ where both $f(x)\to\infty$ and $g(x)\to\infty$ then we have the in determinant form of type $\frac{\infty}{\infty}$

B. L'Hospital's Rule

Suppose that $f(x)$ and $g(x)$ are differentiable, $g'(x)\neq 0$ and that $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$ or that $\lim_{x\to a}\frac{f(x)}{g(x)}=\pm\frac{\infty}{\infty}$ (i.e. we have an in determinant form of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$), then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Examples:

1.) $\lim_{x\to -2}\frac{x^2-4}{x+2}$

2.) $\lim_{x\to -3}\frac{x^2-9}{x+3}$

3.) $\lim_{x\to 1}\frac{\ln x}{x-1}$

4.) $\lim_{x\to 0} \frac{\sin x}{1+\cos x}$

5.) $\lim_{x\to 0} \frac{a^x - 5^x}{9x}$

6.) $\lim_{x\to 0} \frac{1+x-e^x}{3x^2}$

7.) $\lim_{x\to 0} \frac{1-e^{ax}}{x^7} =$

8.) $\lim_{x\to 0} \frac{\sin(10x)}{\sin(bx)} =$

9.) $\lim_{x\to 0+} \frac{\ln x}{x^7}$

So the idea is to be able to get your limit problem into the form: $\lim_{x\to a}\frac{f(x)}{g(x)}$ so you can use **L'Hospital's Rule**

If you have $f(x)\cdot g(x)$ and you check to make sure you get either $0\cdot\infty$ or $\infty\cdot 0$
then you will need to rewrite it first….

you could either rewrite it as $\lim_{x\to a}\frac{f(x)}{1/g(x)}$ *or* $\lim_{x\to a}\frac{g(x)}{1/f(x)}$ *always put the EASY function on the bottom!

10.) $\lim_{x\to 0}\cot 2x\sin 6x$

11.) $\lim_{x\to 0^+} x^4\ln(x)$ =

12.) $\lim_{x\to\infty} x^5 e^{-x^4}$

C. Other "Indeterminate" Forms

$\infty - \infty$ (you will need to rewrite this as either $\frac{0}{0}$ or $\frac{\infty}{\infty}$)
*Try using fractions or factoring

13.) $\lim_{x \to 0} [\csc(ax) - \cot(ax)]$

1.) 0^0 2.) ∞^0 3.) 1^∞	For each of these forms you will need to start by rewriting the problems as y = lim ___. Then you will need to take the **natural log (LN)** of both sides in order to get your exponential function into a multiplication problem using the property of logs. You can then change that into one function divided by another so you can use LH Rule. And lastly, once you get that answer you must set it equal to the **LN y** that you started with on the left hand side. (Phew….It's tough, but you can do it!)

14.) $\lim_{x \to 0^+} x^x$

15.) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x} =$

16.) $\lim_{x\to 0}(1-7x)^{\frac{1}{x}} =$