Sec 4.1 Maximum and Minimum Values

A. Absolute Maximum or Minimum / Extreme Values

A function $f(x)$ has an **Absolute Maximum** at $x = c$ if $f(c) \geq f(x)$ for every point x in the domain.

Similarly, $f(x)$ has an **Absolute Minimum** at $x = c$ if $f(c) \leq f(x)$ for every point x in the domain.

B. Local/Relative Maximum or Minimum Values

A function $f(x)$ has a **Local Maximum** at $x = c$ if $f(c) \geq f(x)$ for every point x that is near c.

A function $f(x)$ has a **Local Minimum** at $x = c$ if $f(c) \leq f(x)$ for every point x that is near c.

C. The Extreme Value Theorem

If $f(x)$ is continuous on a closed interval $[a,b]$ then $f(x)$ attains both a maximum and a minimum value on $[a,b]$

D. Fermat's Theorem

If $f(x)$ has a local maximum or minimum at c, and $f'(c)$ exists, then $f'(c) = 0$.

E. Critical Number

A **critical number** of a function $f(x)$, is a number c in the domain such that $f'(c) = 0$ or $f'(c)$ DNE

If $f(x)$ has a local maximum or minimum at c, then c is critical number of $f(x)$

E. Closed Interval Method

To find **Absolute Maximum or Minimum** of a continuous function $f(x)$ on a closed interval $[a,b]$:

1. Find the values of $f(x)$ at the critical numbers of $f(x)$ in (a,b).
2. Find the values of $f(x)$ at the endpoints a and b of the interval.
3. The largest of the values of step 1 and 2 is the **Absolute Maximum**
4. The smallest of the values of step 1 and 2 is the **Absolute Minimum**

Examples:

1.) Find the critical numbers for the following functions

a. $f(x) = 2x^2 + 9x - 2$

b. $f(x) = -2x^3 + 33x^2 - 60x + 11$

c. $f(x) = x^{4/5}(x-5)$

d. $f(x) = (6x-2)e^{-6x}$

2.) Consider the function $f(x) = 3x^2 - 6x + 8, \quad 0 \leq x \leq 10$.

The absolute maximum value of $f(x)$ (on the given interval) is __________
and this occurs at x equals__________

and the absolute minimum of $f(x)$ (on the given interval) is__________
and this occurs at x equals__________

3.) Consider the function $f(x) = 2x^3 + 18x^2 - 162x + 9$ on the interval $-9 \leq x \leq 4$.
Find the critical numbers and absolute minimum and maximum values.

4.) Consider the function $f(x) = x + 2\cos x$ on the interval $0 \le x \le \pi$. Find the absolute maximum and minimum of the function.

5.) Choose the best reason that the function $f(x) = x^{91} + x^{25} + x^7 + 13$ has neither a local maximum nor a local minimum.

(a) The function f(x) is always positive.
(b) The derivative f ′ (x) is always negative.
(c) The derivative f ′ (x) is always positive.
(d) The highest power of x in f(x) is odd.