Sec 4.2 The Mean Value Theorem

A. Rolle's Theorem

Let $f(x)$ be a function such that $f(x)$ is continuous on $[a,b]$, $f(x)$ is differentiable on (a,b) and $f(a)=f(b)$
Then there is a number c in (a,b) such that $f'(c)=0$

B. The Mean Value Theorem

Let $f(x)$ be a function such that $f(x)$ is continuous on $[a,b]$ and $f(x)$ is differentiable on (a,b) .
Then there is a number c in (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$
Or equivalently $f(b)-f(a)=f'(c)\cdot(b-a)$

C. Constant Theorem

If $f'(x)=0$ for all x in an interval (a,b) then $f(x)$ constant on (a,b).

D. Corollary

If $f'(x)=g'(x)$ for all x in an interval (a,b) then $f-g$ is constant on (a,b) (i.e. $f(x)=g(x)+c$)

Examples

1.) Consider the function $f(x)=x^2-4x+1$ on the interval $[0,4]$. Verify that this function satisfies the three hypotheses of Rolle's Theorem on the interval.

$f(x)$ is_______________on $[0,4]$; $f(x)$ is________________on $(0,4)$; and $f(0)=f(4)=$____________

Then by Rolle's theorem, there exists a c such that $f'(c)=0$. Find the value c.

2.) Consider the function $f(x)=3x^2+5x+11$ on the interval $[-3,6]$. Find the average or mean slope of the function on this interval, i.e.

$$\frac{f(6)-f(-3)}{6-(-3)} =$$

By the Mean Value Theorem, we know there exists a c in the open interval $(-3,6)$ such that $f'(c)$ is equal to this mean slope. For this problem, there is only one c that works. $c =$

3.) By applying Rolle's Theorem, check whether it is possible that the function $f(x) = x^5 + x - 11$ has two real roots.

Possible or impossible?

Your reason is that if $f(x)$ has two real roots then by Rolle's theorem: $f'(x)$ must be ___________

at certain value of x between these two roots, but $f'(x)$ is always negative ,positive, or zero______________

4.) Suppose $f(x)$ is continuous on [2,8] and $2 \le f'(x) \le 8$ for all x in (2,8) . Use the Mean Value Theorem to estimate $f(8) - f(2)$.

_____ $\le f(8) - f(2) \le$ _____