Sec 4.3 Derivatives and the Shape of Graphs

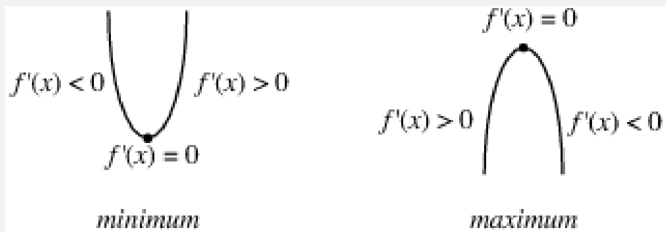
A. The First Derivative

Increasing/Decreasing Test

- If $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval
- If $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval

A **critical number** of a function $f(x)$, is a number c in the domain such that $f'(c) = 0$ or $f'(c)$ DNE

If $f(x)$ has a local maximum or minimum at c, then c is critical number of $f(x)$



The First Derivative Test

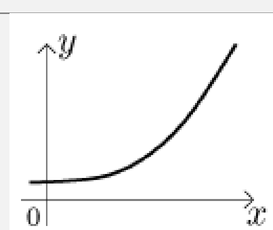
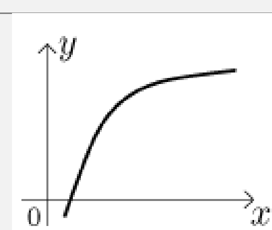
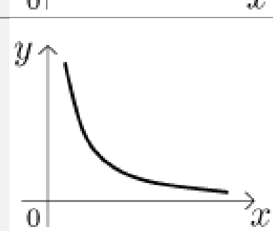
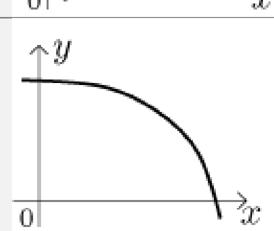
Suppose c is a critical number of a continuous function $f(x)$

- If $f'(x)$ changes from positive to negative at c, then $f(x)$ has a local maximum at c
- If $f'(x)$ changes from negative to positive at c, then $f(x)$ has a local minimum at c
- If $f'(x)$ does not change sign at c, then $f(x)$ has no local maximum or minimum at c

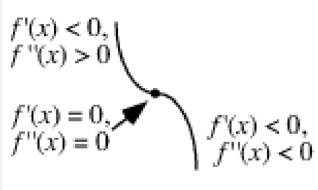
B. The Second Derivative

Concavity

- If $f''(x) > 0$ on an interval, then $f(x)$ is concave up on that interval
- If $f''(x) < 0$ on an interval, then $f(x)$ is concave down on that interval

	Concave Up	Concave Down
Increasing Slope		
Decreasing Slope		

An **inflection point** of a function $f(x)$, is a point at which the curvature (second derivative) changes sign. The curve changes from being concave upwards (positive curvature) to concave downwards (negative curvature), or vice versa.



inflection point

The Second Derivative Test

Suppose that $f''(x)$ is continuous at c

- If $f'(x)=0$ and $f''(x)>0$ then $f(x)$ has a local minimum at c.
- If $f'(x)=0$ and $f''(x)<0$ then $f(x)$ has a local maximum at c.

Example:

1.) $f(x)=2x^3-3x^2-12x$

a.) Find the critical points and the intervals on increase and decrease.
b.) State whether each critical point is a maximum or a minimum.
c.) Find the inflection points and the intervals on concavity.
d.) Sketch the graph and verify your results.

2.) $g(x) = x + 2\cos(x)$ on $0 \le x \le 2\pi$

a.) Find the critical points and the intervals on increase and decrease.
b.) State whether each critical point is a maximum or a minimum.
c.) Find the inflection points and the intervals on concavity.
d.) Sketch the graph and verify your results.

3.) $h(x)=\dfrac{e^x}{e^x+8}$

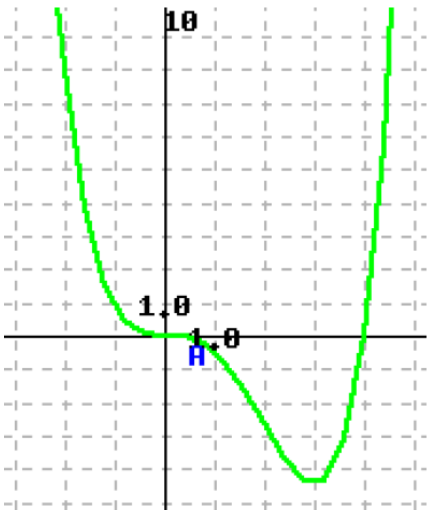
a.) Find the critical points and the intervals on increase and decrease.
b.) State whether each critical point is a maximum or a minimum.
c.) Find the inflection points and the intervals on concavity.
d.) Sketch the graph and verify your results..

4.) Suppose that $f''(x)$ is continuous on $(-\infty,\infty)$.

a.) If $f'(5)=0$ and $f''(5)=6$, then f has a local_______________ at $x=5$.

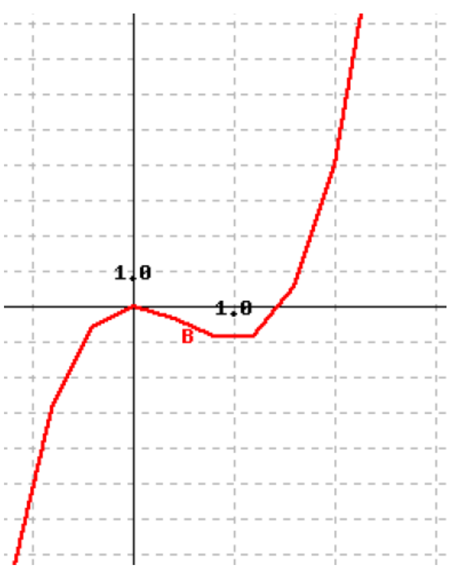
b.) If $f'(19)=0$ and $f''(19)=-6$, then f has a local_______________ at $x=19$.

5.) Given the graph of $f(x)$, determine whether the following conditions are true.



1. $f'(3)=0$
2. $f''(x)>0$ if $0<x<2$
3. $f''(x)>0$ if $x<0$
4. $f'(x)\le 0$ if $x<3$
5. $f'(0)=0$

6.) Given the graph of $f'(x)$, determine whether the following conditions are true.



1. f is concave downward on the interval $(-\infty,0)$
2. f has a local maximum at x = 0
3. f is decreasing on the interval $(-\infty,1.5)$
4. f has an inflection point at x = 0
5. f is decreasing on the interval $(0,1)$
6. f is increasing on the interval $(1.5,\infty)$

7.) Find a cubic function $f(x) = ax^3 + cx^2 + d$ that has a local maximum value of 8 at $x = -2$ and a local minimum value of 6 at $x = 0$.