Sec 4.4 Curve Sketching

A. Guidelines for sketching a curve

1. Domain
2. Intercepts (x-intercepts and y-intercepts)
3. Symmetry (Odd, even or periodic functions)
4. Asymptotes
5. Intervals of Increase and Decrease
6. Maximum and Minimum Values
7. Intervals of Concavity

Example: Sketch the curve using the guidelines 1 – 7.

1.) $f(x)=\dfrac{x^2}{x^2-4}$

1. Domain

2. Intercepts (x-intercepts and y-intercepts)

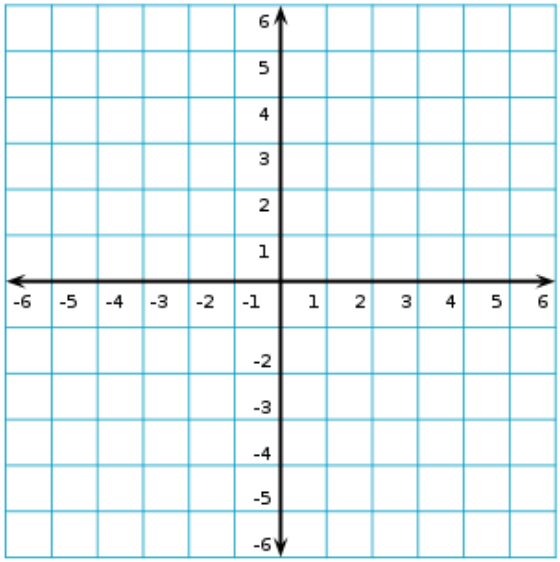
3. Asymptotes

4. Intervals of Increase and Decrease

5. Maximum and Minimum Values

6. Intervals of Concavity

7. Inflection Points



2.) $f(x)=3\cos(x)-\cos^3(x)$ on $0\le x\le 2\pi$

1. Domain
2. Intercepts (x-intercepts and y-intercepts)
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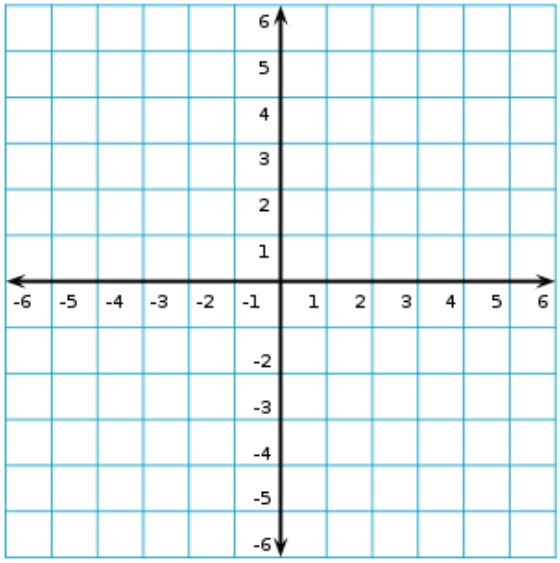


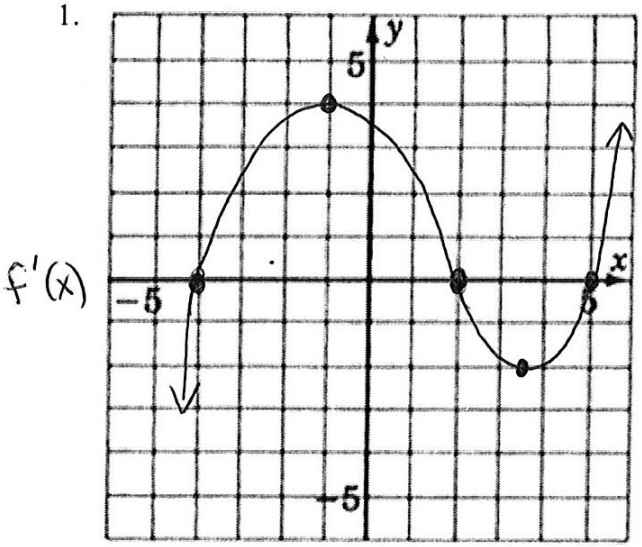
B. Guidelines for sketching a function given a sketch of it's derivative.

1. Find all intervals where the function is increasing and decreasing
2. Find all intervals where the function is concave up and concave down
3. Sketch a function that has these characteristics (there are many graphs possible)

f(x) : max and mins ↔ f '(x): roots
f(x): inflection points ↔ f '(x): max and mins
f(x): concave up ↔ f '(x): increasing
f(x): concave down ↔ f '(x): decreasing
f(x): increasing ↔ f '(x) > 0
f(x): decreasing ↔ f '(x) < 0

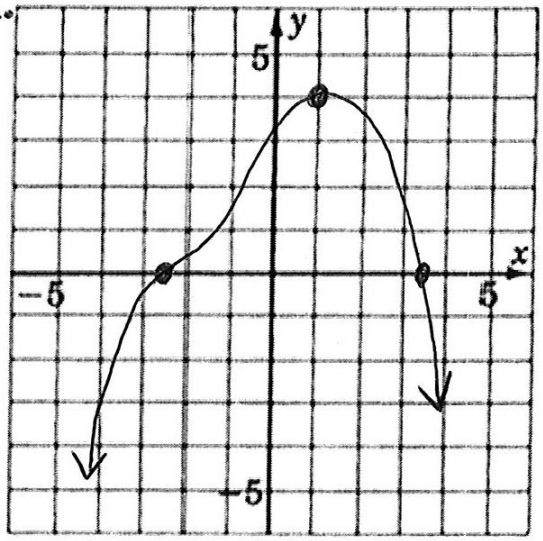
For each of the following state where *f(x)* is increasing, decreasing, has max and mins, concave up and down, inflection points and sketch *f(x)*.

1.

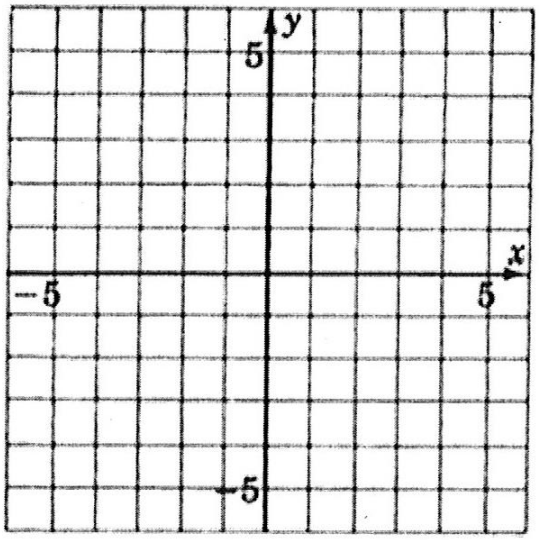


f'(x)

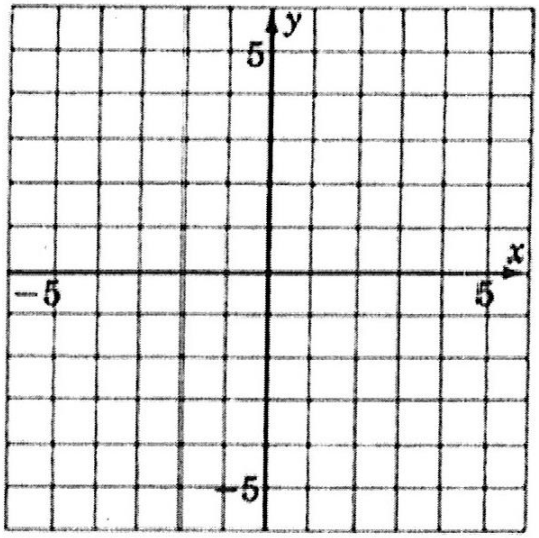
2.



f'(x)



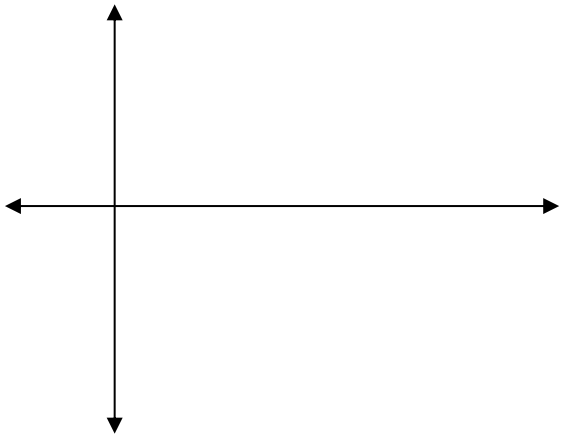
f(x)



f(x)

Sketch the graph of a function, *f(x)*, that satisfies all of the given conditions.

3. *f ’(0) = f ’(2) = f ’(4) = 0*
   f ’(x) > 0 if x < 0 or 2 < x < 4
   f ’(x) < 0 if 0 < x < 2 or x > 4
   f ”(x) > 0 if 1 < x < 3
   f ”(x) < 0 if x < 1 or x > 3



4. *f(0) = 0, f ’(-2) = f ’(1) = f ’(9) = 0*
   $\lim_{x\to\infty} f(x) = 0 \quad \lim_{x\to 6} f(x) = -\infty$
   f ’(x) < 0 on (-∞,-2),(1,6) and (9,∞)
   f ’(x) > 0 on (-2,1) and (6,9)
   f ”(x) > 0 on (-∞,0) and (12,∞)
   f ”(x) < 0 on (0,6) and (6,12)