Key

Homework Set 10

(sect 6.4: Integration with Tables)

Use the Table of Integrals found at the back of your textbook (or on the class website) and an appropriate u-substitution to evaluate the given integral. State which rule you used.

1. $\int_0^1 2x\cos^{-1}x\,dx$

$= 2\int_0^1 x\cos^{-1}x\,dx$

$= 2\left[\frac{2x^2-1}{4}\cos^{-1}x - \frac{x\sqrt{1-x^2}}{4}\right]\Big|_0^1$

$= \left(\frac{2-1}{2}\cos^{-1}(1) - 0\right) - \left(\frac{-1}{2}\cos^{-1}(0) - 0\right)$

$= 0 + \frac{1}{2}\cdot\pi/2 = \pi/4$

table rule #91

$\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$

2. $\int \frac{dx}{x^2\sqrt{4x^2+9}}$

$= \int \frac{\frac{1}{2}du}{\frac{1}{4}u^2\sqrt{u^2+9}} = 2\int \frac{du}{u^2\sqrt{u^2+9}}$

$= 2\left[-\frac{\sqrt{u^2+9}}{9u} + C\right]$

$= -\frac{2\sqrt{4x^2+9}}{18x} + C$

$= -\frac{\sqrt{4x^2+9}}{9x} + C$

table rule #28

$u^2 = 4x^2 \rightarrow u = 2x$

$du = 2\,dx \rightarrow \frac{1}{2}du = dx$

$x^2 = \frac{1}{4}u^2$ & $a^2 = 9 \rightarrow a = 3$

$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$

3. $\int \sin^2 x\cos x\ln(\sin x)\,dx$

$= \int u^2 \ln u\,du$

$= \frac{u^3}{9}[3\ln u - 1] + C$

$= \frac{1}{3}(\sin x)^3\ln(\sin x) - \frac{1}{9}(\sin x)^3 + C$

$u = \sin x$

$du = \cos x\,dx$

table rule #101

$\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

$n = 2$