Key

Homework Set 12

(sect 8.1: Sequences)

Find the next term in the sequence.

1. $\{0, 1, 2, 3, 4, \ldots\}$ $a_6 = 5$

   $+1$

   $a_n = a_{n-1} + 1$

2. $\{6, -18, 54, -162, 486, \ldots\}$ $a_6 = -1458$

   $\times(-3)$

   $a_n = -3a_{n-1}$

3. $\{1, 8, 27, 64, 125, \ldots\}$ $a_6 = 6^3 = 216$

   $a_n = n^3$

4. $\{2, 3, 5, 8, 12, \ldots\}$ $a_6 = 17$

   1 2 3 4 5

5. $\{1, 3, 7, 13, 21, \ldots\}$ $a_6 = 31$

   2 4 6 8 10

Find a closed formula for the general term a_n, assuming that the pattern of the first terms continues. Be sure you indicate whether you assume that the sequence starts at a_0 or a_1 or at some other index. (Note: a recursive formula for the general term a_n will only give you have credit.)

6. $\{7, 11, 15, 19, 23, \ldots\}$

   $a_n = 4n + 3$ if $a_1 = 7$ $(a_n = a_{n-1} + 4)$

   $a_n = 4n + 7$ if $a_0 = 7$

   $a_1 = 7$
   $a_2 = 7 + 4$
   $a_3 = 7 + 2 \cdot 4$
   $\vdots$
   $a_n = 7 + (n-1)4 = 4n + 3$

   $a_0 = 7$
   $a_1 = 7 + 4$
   $a_2 = 7 + 2 \cdot 4$
   $\vdots$
   $a_n = 7 + 4n$

7. $\left\{\frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \ldots\right\}$

   $a_n = \dfrac{n+1}{n^2}$ if $a_2 = 3/4$

   $a_n = \dfrac{n+2}{(n+1)^2}$ if $a_1 = 3/4$

8. $\left\{1, -\frac{2}{7}, \frac{4}{49}, -\frac{8}{343}, \ldots\right\}$

   $a_n = (-2/7)^{n-1}$ if $a_1 = 1$ $(a_n = a_{n-1} \cdot (-\frac{2}{7}))$

   OR $a_n = (-2/7)^n$ if $a_0 = 1$

   $a_1 = 1$
   $a_2 = -2/7$
   $a_3 = (-2/7)^2$
   $\vdots$
   $a_n = (-2/7)^{n-1}$

   $a_0 = 1$
   $a_1 = -2/7$
   $a_2 = (-2/7)^2$
   $\vdots$
   $a_n = (-2/7)^n$

List the first 4 terms of the given sequence. Assume that the first term is a_1.

9. $a_k = k + \frac{1}{k}$ $\quad a_1 = 2, \quad a_2 = 5/2, \quad a_3 = 10/3, \quad a_4 = 17/4$

10. $a_n = \frac{(-1)^n 2^n}{4-3n}$ $\quad a_1 = -2, \quad a_2 = -2, \quad a_3 = 8/5, \quad a_4 = -2$

Determine whether the sequence converges or diverges. If it converges, find its limit.

11. $a_n = \frac{3n^2-1}{2n^3+1}$ $\quad \lim_{n\to\infty} \frac{3n^2-1}{2n^3+1} = \lim_{n\to\infty} \frac{3}{2n} = 0$

converges to 0

12. $a_n = \frac{\sqrt{n}-1}{5\sqrt{n}}$ $\quad \lim_{n\to\infty} \frac{\sqrt{n}-1}{5\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}}{5\sqrt{n}} = 1/5$

converges to $1/5$

13. $a_k = 2^k 3^{-k}$ $\quad \lim_{k\to\infty} \frac{2^k}{3^k} = \lim_{k\to\infty} (2/3)^k = 0$

converges to 0

14. $a_k = \frac{(k+1)!}{7k!}$ $\quad \lim_{k\to\infty} \frac{(k+1)!}{7k!} = \lim_{k\to\infty} \frac{k+1}{7} = \infty$

diverges

15. $a_n = \left(1+\frac{2}{n}\right)^n$

$\lim_{n\to\infty} (1+2/n)^n = e^2$

converges to e^2

16. Determine whether the sequence $a_n = \frac{2n^2}{n^2+1}$ is increasing, decreasing, or not monotonic. Is the sequence bounded?

$a_n = \frac{2n^2}{n^2+1}$ is increasing

b/c: $2n^2 < 2(n^2+1)$

$2n^2(2n+1) < 2(n^2+1)(2n+1)$

$2n^2(2n+1) + 2n^2(n^2+1) < 2n^2(n^2+1) + 2(n^2+1)(2n+1)$

$2n^2(n^2+2n+2) < 2(n^2+1)(n^2+2n+1)$

$a_n = \frac{2n^2}{n^2+1} < \frac{2n^2+4n+2}{n^2+2n+2} = \frac{2(n+1)^2}{(n+1)^2+1} = a_{n+1}$

$a_0 = 0$ & $\lim_{n\to\infty} \frac{2n^2}{n^2+1} = 2$

So for any n, $0 \le a_n \le 2$

ie: $\{a_n\}$ is bounded