Key

Homework Set 14

(sect 8.3: Integral & P-series Tests)

Use the integral test or the p-series test to determine whether each series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{3}{n^{1/3}}$$

diverges

$p = 1/3 < 1$

$$\sum_{k=1}^{\infty} \frac{1}{k^5}$$

Converges

$p = 5 > 1$

$$\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^2+5}}$$

diverges

$$\int_0^{\infty} \frac{x}{\sqrt{x^2+5}}\,dx = \frac{1}{2}\int_0^{\infty} (x^2+5)^{-1/2} \cdot 2x\,dx$$

$$= \frac{1}{2} \cdot 2(x^2+5)^{1/2}\Big|_0^{\infty} = \sqrt{(x^2+5)}\Big|_0^{\infty} = \text{"}\infty\text{"} - \sqrt{5} = \infty$$

$$\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$$

diverges

$$\int_2^{\infty} \frac{\ln x}{x}\,dx = \int_{\ln 2}^{\infty} u\,du = \frac{1}{2}u^2\Big|_{\ln 2}^{\infty} = \text{"}\infty\text{"} - \frac{(\ln 2)^2}{2} = \infty$$

$u = \ln x$

$du = \frac{1}{x}dx$

x	u
∞	∞
2	$\ln 2$

$$\sum_{k=4}^{\infty} \frac{7}{k\sqrt{k}} = \sum_{k=4}^{\infty} \frac{7}{k^{3/2}}$$

converges

$p = 3/2 > 1$

$$\sum_{n=1}^{\infty} ne^{-3n^2}$$

Converges

$$\int_1^{\infty} xe^{-3x^2}\,dx = -\frac{1}{6}\int_1^{\infty} -6xe^{-3x^2}\,dx = -\frac{1}{6}e^{-3x^2}\Big|_1^{\infty}$$

$$= \text{"}\frac{-1}{\infty}\text{"} + \frac{1}{6}e^{-3} = 0 + \frac{1}{6}e^{-3} = \frac{1}{6e^3}$$