Key

Homework Set 16

(sect 8.4: Ratio Test)

Use the ratio test to determine whether each series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{n5^n}{n!}$$

Converges

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{(n+1)5^{n+1}}{(n+1)!} \cdot \frac{n!}{n5^n}\right]$$

$$= \lim_{n\to\infty} \frac{5(n+1)}{n(n+1)} = \lim_{n\to\infty} \frac{5}{n} = 0 < 1$$

$$\sum_{n=1}^{\infty} \frac{n^2-1}{4^n}$$

converges

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{(n+1)^2-1}{4^{n+1}} \cdot \frac{4^n}{n^2-1}\right]$$

$$= \lim_{n\to\infty} \frac{n^2+2n}{4(n^2-1)} = \lim_{n\to\infty} \frac{n^2+2n}{4n^2-4} = 1/4 < 1$$

$$\sum_{n=2}^{\infty} ne^{-n} = \sum_{n=2}^{\infty} \frac{n}{e^n}$$

converges

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n}\right]$$

$$= \lim_{n\to\infty} \frac{n+1}{en} = 1/e < 1$$

$$\sum_{k=1}^{\infty} \frac{k^k}{k!}$$

diverges

$$\lim_{k\to\infty} \frac{(k+1)^{k+1}}{(k+1)!} \cdot \frac{k!}{k^k} = \lim_{k\to\infty} \frac{(k+1)^k}{k^k}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^k = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^k = e > 1$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{n+2}}{2^{n+1}(n-1)^2}$$

Diverges

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{(-1)^{n+1} \cdot 3^{n+3}}{2^{n+2} \cdot n^2} \cdot \frac{2^{n+1}(n-1)^2}{(-1)^n \cdot 3^{n+2}}\right]$$

$$= \lim_{n\to\infty} \frac{(-1) \cdot 3 \cdot (n-1)^2}{2n^2} = \lim_{n\to\infty} \frac{-3n^2+6n-3}{2n^2} = -3/2$$

So, $L = -3/2 < -1$