Key

Homework Set 19

(sect 8.6: Functions as Power Series)

Find a power series representation for each function. Be sure to indicate for which x values your power series is valid. You may want to use the following fact:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}, \quad |x| < 1$$

1. $f(x) = \frac{1}{1-x^4}$

$$\frac{1}{1-(x^4)} = 1 + (x^4) + (x^4)^2 + \ldots = \sum_{n=0}^{\infty} (x^4)^n = \sum_{n=0}^{\infty} x^{4n}$$

and $|x^4| < 1 \Rightarrow |x| < 1$

2. $f(x) = \frac{1}{1+2x^3}$

$$\frac{1}{1-(-2x^3)} = 1 + (-2x^3) + (-2x^3)^2 + \ldots = \sum_{n=0}^{\infty} (-2x^3)^n = \sum_{n=0}^{\infty} (-2)^n x^{3n}$$

and $|-2x^3| < 1 \Rightarrow |x| < \frac{1}{\sqrt[3]{2}}$

3. $f(x) = \frac{7x}{1-x^4}$

$$\frac{7x}{1-x^4} = 7x \cdot \frac{1}{1-x^4} = 7x + 7x(x^4) + 7x(x^4)^2 + \ldots = \sum_{n=0}^{\infty} 7x \cdot x^{4n} = \sum_{n=0}^{\infty} 7x^{4n+1}$$

$|x| < 1$

4. $f(x) = \frac{x^3}{(1-x)^2}$ Hint: use differentiation

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{n=0}^{\infty} x^n$$ ↘ derivative

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \ldots = \sum_{n=0}^{\infty} (n+1)x^n$$

$$\frac{x^3}{(1-x)^2} = x^3 + 2x^4 + 3x^5 + \ldots = \sum_{n=0}^{\infty} (n+1)x^{n+3}, \quad |x| < 1$$

5. $f(x) = \ln(1-x)$ Hint: use integration

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{n=0}^{\infty} x^n$$

$$-\ln(1-x) = x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \ldots = \sum_{n=1}^{\infty} \tfrac{1}{n} x^n$$

$$\ln(1-x) = -x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \ldots = \sum_{n=1}^{\infty} \tfrac{-1}{n} x^n, \quad |x| < 1$$