Homework Set 2

(sect 5.1 & 5.2: Riemann Sums)

Unless otherwise indicated, give your answers correct to 6 decimal places.

1. For both parts (a) and (b), estimate the area under the curve $f(x) = 2/x$ from $x = 2$ to $x = 4$ using 4 approximating rectangles and the indicated endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   a. Right endpoints.

   b. Left endpoints.

2. For both parts (a) and (b), estimate the area under the curve $f(x) = e^{-x^2}$ on $-2 \leq x \leq 2$ using the indicated number of approximating rectangles and midpoints. Sketch the graph and the rectangles. Does your estimate improve?
   a. $n = 4$

   b. $n = 8$

3. If $f(x) = \ln x - 1, 1 \leq x \leq 4$, evaluate the right Riemann Sum with $n = 6$. What does the Riemann Sum represent? Illustrate with a diagram.

4. Find the Riemann Sum for $f(x) = x + x^2, -2 \leq x \leq 0$, if the partition points are $-2, -1.5, -1, -0.7, -0.4, 0$ and the sample points are left endpoints. What is the max Δx_i?

5. Consider the curve $f(x) = x^3$ on the interval $0 \leq x \leq 1$.
   a. Find an expression for the area under the curve as the limit of a right Riemann sum.

$\Delta x =$

$x_i =$

$R_n =$

$Area =$

   b. Compute the limit found in part (a). [Hint: $1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$]

For questions 6 – 8, express the limit as a definite integral on the given interval.

6. $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i - \sin x_i)\Delta x, \quad [1,5]$

7. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i}{1+e^{x_i}} \Delta x, \quad [0,3]$

8. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{i}{n}\right)^4 \cdot \frac{1}{n}, \quad [0,1]$

For questions 9 – 10, express the definite integral as a limit of right Riemann Sums.

9. $\int_{-1}^{2} (1 - 5x)\, dx$

10. $\int_{3}^{10} \frac{\ln x}{x} dx$