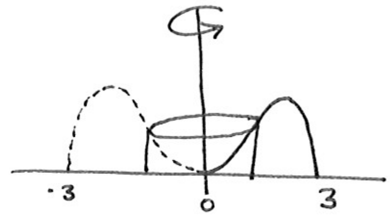
Key

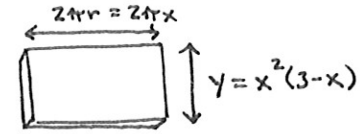
Homework Set ~~24~~ 22

(sect 7.2 & 7.3: Volume)

1. Let S be the solid obtained by rotating the region bounded by $y = x^2(3 - x)$ and $y = 0$ about the y-axis. Find the volume V of S by using the shell method. It may be helpful to draw a picture of the situation. Explain why it would have been awkward to use slices to find V.

$$V = \int_0^3 2\pi x \cdot x^2(3-x)\,dx$$
$$= 2\pi \int_0^3 3x^3 - x^4\,dx$$
$$= 2\pi \left(\tfrac{3}{4}x^4 - \tfrac{1}{5}x^5\right)\Big|_0^3$$
$$= \frac{243\pi}{10} \quad \text{OR} \quad 76.3407$$





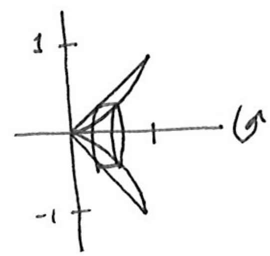
Slices would require washers, but we only have 1 function

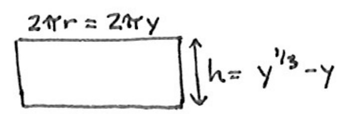
2. Consider the region bounded by $y = x^3$ and $y = x$ in the first quadrant. Find the volume of the solid obtained by rotating this region about the x-axis.

$y = x^3 \Rightarrow x = y^{1/3}$

   a. Use cylindrical shells to find this volume.

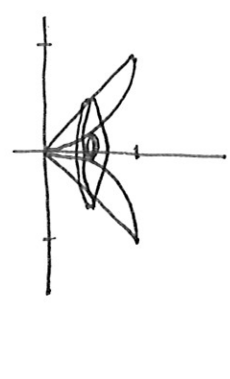
$$V = \int_0^1 2\pi y\left(y^{1/3} - y\right)dy$$
$$= 2\pi \int_0^1 y^{4/3} - y^2\,dy$$
$$= 2\pi \left(\tfrac{3}{7}y^{7/3} - \tfrac{1}{3}y^3\right)\Big|_0^1$$
$$= \frac{4\pi}{21} \quad \text{OR} \quad .5983986$$





   b. Use slices to find this volume.

$$V = \int_0^1 \pi(x)^2 - \pi(x^3)^2\,dx$$
$$= \pi \int_0^1 x^2 - x^6\,dx$$
$$= \pi\left(\tfrac{1}{3}x^3 - \tfrac{1}{7}x^7\right)\Big|_0^1$$
$$= 4\pi/21 \quad \text{OR} \quad .5983986$$



$r = x^3$

$R = x$

   c. Which method do you prefer? Why?

3. Use the shell method to find the volume of the solid obtained by rotating the region bounded by $y = 5x - x^2$ and $y = x$ about the y-axis. (Hint: draw a picture of the situation and label where the height and radius of the cylinders are located.)

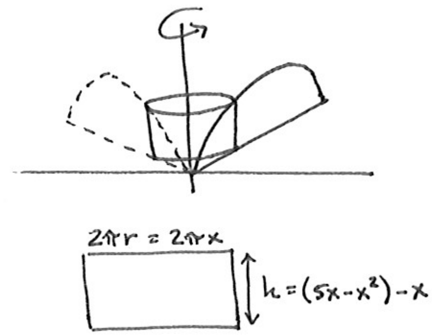
$$V = 2\pi \int_0^4 x(4x - x^2)\,dx$$

$$= 2\pi \int_0^4 4x^2 - x^3\,dx$$

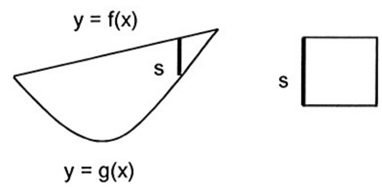
$$= 2\pi\left(\tfrac{4}{3}x^3 - \tfrac{1}{4}x^4\right)\Big|_0^4$$

$$= 128\pi/3$$

$$\text{OR } = 134.0412866$$



4. The base of a certain solid is the area bounded above by the line $y = f(x) = x + 2$ and below by the curve $y = g(x) = x^2 - 4$. The cross-sections perpendicular to the x-axis are squares. (see figures to the right)



Use the formula $V = \int_a^b A(x)\,dx$ to find the volume of the solid.

The side of the square cross-section is a function of x, given by $s(x) = (x+2) - (x^2 - 4) = 6 + x - x^2$

$$x + 2 = x^2 - 4$$

$$0 = x^2 - x - 6 = (x-3)(x+2) \Rightarrow x = -2, 3$$

$a = -2$

$b = 3$

$A(x) = s^2 = (6 + x - x^2)^2$

Thus, the volume of the solid is $V =$

$$\int_{-2}^{3} (6 + x - x^2)^2\,dx$$

$$= \int_{-2}^{3} (x^4 - 2x^3 - 11x^2 + 12x + 36)\,dx$$

$$= \tfrac{1}{5}x^5 - \tfrac{1}{2}x^4 - \tfrac{11}{3}x^3 + 6x^2 + 36x \Big|_{-2}^{3}$$

$$= 625/6$$

$$\text{OR } 104.1\overline{6}$$