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Homework Set

(sect 7.4: Arc Length)

1. Use the arc length formula to find the length of the curve $y = 5 - 2x$ on $1 \le x \le 3$. Check your answer by computing the length of the curve using the distance formula.

$$L = \int_a^b \sqrt{(x')^2 + (y')^2} = \int_1^3 \sqrt{1 + (-2)^2}\, dx = \sqrt{5}\, x \Big|_1^3 = 2\sqrt{5}$$

$$D = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2} = \sqrt{(-1-3)^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Find the length of the curve along the given interval.

2. $y = \ln(\sec x)$, $0 \le x \le \pi/4$

$$y' = \frac{\sec x \tan x}{\sec x} = \tan x$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2} + 1) - \ln 1 = \ln(\sqrt{2} + 1) \overset{\text{OR}}{=} .88137$$

3. $x = 1 + 2y^{3/2}$, $0 \le y \le 1$

$x' = 3y^{1/2}$

$u = 1 + 9y$

$du = 9dy$

x	u
1	10
0	1

$$L = \int_0^1 \sqrt{1 + 9y}\, dy = \int_1^{10} \frac{1}{9} u^{1/2}\, du$$

$$= \frac{1}{9} \cdot \frac{2}{3} u^{3/2} \Big|_1^{10} = \frac{2}{27}\left[10^{3/2} - 1\right] = 2.26835$$

4. $x = 2 + t^3$, $y = 1 + 2t^2$, $0 \le t \le 2$

$x' = 3t^2$

$y' = 4t$

$u = 9t^2 + 16$

$du = 18t\, dt$

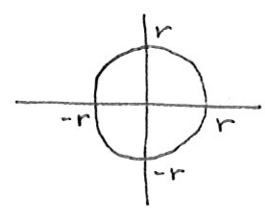
x	u
2	52
0	16

$$L = \int_0^2 \sqrt{9t^4 + 16t^2}\, dt = \int_0^2 t\sqrt{9t^2 + 16}\, dt$$

$$= \int_{16}^{52} \frac{1}{18} u^{1/2}\, du = \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \Big|_{16}^{52} = \frac{1}{27} u^{3/2} \Big|_{16}^{52}$$

$$= 11.517679$$

5. Sketch the graph of the curve given by the parametric equation: $x = r \cdot \cos t$, $y = r \cdot \sin t$, $0 \le t \le 2\pi$ where r is any positive number. Use the Arc Length Formula to calculate the length of this circle. Show all of your work.



$x' = \underline{-r\sin t}$ $\quad$ $y' = \underline{r\cos t}$

$$\text{Arc Length } L = \int_0^{2\pi} \sqrt{(-r\sin t)^2 + (r\cos t)^2}\, dt$$

$$= r\int_0^{2\pi} \sqrt{(\sin t)^2 + (\cos t)^2}\, dt \quad \text{by factoring out } r^2$$

$$= r\int_0^{2\pi} 1\, dt = rt\Big|_0^{2\pi} = r \cdot 2\pi = 2r\pi$$

Is this the expected value for the circumference of the circle?

yes