Key

# Homework Set ~~2~~ 24

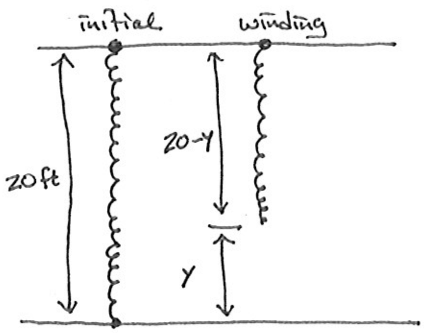
(sect 7.5: Work)

1. A particle is moved along the x-axis by a force that measures $5/(x+1)^2$ pounds at a point $x$ feet from the origin. Find the work done in moving the particle from the origin to a distance of 9 feet.

$$\Delta W = F \cdot \Delta d = \frac{5}{(x+1)^2} \cdot \Delta x \quad \text{for } 0 \le x \le 9$$

$$W = \int_0^9 \frac{5}{(x+1)^2}\,dx = \int_0^5 5(x+1)^{-2}\,dx = -5(x+1)^{-1}\Big|_0^5 = \frac{-5}{6} + \frac{5}{1} = \frac{25}{6} \text{ ft-lb}$$

2. Consider a 20-foot chain hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up the chain until the bottom of the chain is 10 foot above the ground if the chain weighs 3 lb/ft.



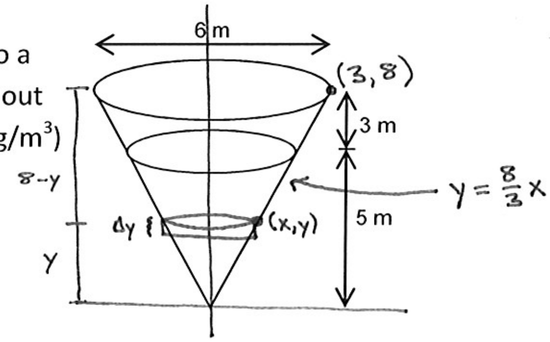

$$\Delta W = \Delta F \cdot d$$
$$= \Delta F \cdot y$$

$y$ = how far we've wound the chain so far. $(0 \le y \le 10)$

$$\Delta F = \text{weight}$$
$$= (\text{unit weight})(\text{length})$$
$$= (3 \text{ lb/ft})(\Delta y \text{ ft}) = 3\,\Delta y \text{ lb}$$
$$\Delta W = 3y\,\Delta y \text{ ft-lb}$$

$$W = \int_0^{10} 3y\,dy = 150 \text{ ft-lb}$$

3. The right cone-shaped tank to the right is filled with water to a height of 5m. Find the work required to pump all the water out of the tank. (Use the fact that the density of water is 1000 kg/m$^3$)




$$\Delta W = \Delta F \cdot d$$

$$d = 8 - y \quad \& \quad 0 \le y \le 5$$

$$\Delta V = \pi r^2 \Delta y = \pi x^2 \Delta y$$
$$= \pi\left(\tfrac{3}{8}y\right)^2 \Delta y$$
$$= \frac{9\pi}{64} y^2 \Delta y \text{ m}^3$$

$$\Delta m = \Delta V \cdot \rho$$
$$= \left(\frac{9\pi}{64} y^2 \Delta y \text{ m}^3\right)\left(1000 \text{ kg/m}^3\right)$$
$$= \frac{1125\pi}{8} y^2 \Delta y \text{ kg}$$

$$\Delta F = \Delta m \cdot g$$
$$= \left(\frac{1125\pi}{8} y^2 \Delta y \text{ kg}\right)\left(9.8 \text{ m/s}^2\right) = \frac{11025\pi}{8} y^2 \Delta y \text{ N}$$

$$\Delta W = \left(\frac{11025\pi}{8} y^2 \Delta y \text{ N}\right)\left(8 - y \text{ m}\right)$$

$$W = \int_0^5 \frac{11025\pi}{8}(8-y)y^2\,dy = \frac{7809375\pi}{32} \text{ J} = 766683.598 \text{ J}$$

| inches | ft |
| --- | --- |
| 15 | 5/4 |
| 4 | 1/3 |
| 7 | 7/12 |

4. A force of 5 pounds compresses a 15-inch spring a total of 4 inches. How much work is done in compressing the spring 7 inches? (Use the fact that Hook's Law is $f(x) = kx$.)

$f(x) = kx$

$5 = k(1/3)$

$15 = k$

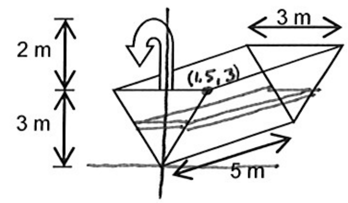
$\Rightarrow f(x) = 15x$

$\Delta W = F \cdot \Delta d = 15x \cdot \Delta x$ & $0 \le x \le 7/12$

$$W = \int_0^{7/12} 15x \, dx$$

$$= \frac{15}{2}x^2 \Big|_0^{7/12} = \frac{245}{96} \text{ ft-lb}$$

5. The tank shown to the right is full of water. Find the work required to pump water out of the outlet. You may assume that the triangular sides are isosceles. (Use the fact that the density of water is 1000 kg/m$^3$)




$\Delta V = 2x \cdot 5 \cdot \Delta y$

$= 2(y/2) \cdot 5 \cdot \Delta y = 5y \, \Delta y$ m$^3$

line: $y = mx$, $m = \frac{3}{1.5} = 2$

$y = 2x$

dist $= 5 - y$

& $0 \le y \le 3$

$\Delta m = \Delta V \cdot \rho$

$= (1000 \text{ kg/m}^3)(5y \, \Delta y \text{ m}^3) = 5000y \, \Delta y$ kg

$\Delta F = \Delta m \cdot g$

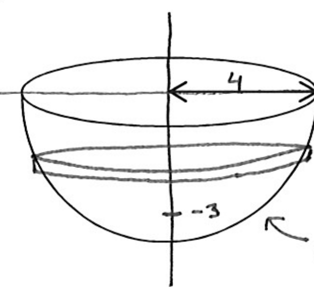
$= (9.8 \text{ m/s}^2)(5000y \, \Delta y \text{ kg}) = 49000y \, \Delta y$ N

$\Delta W = \Delta F \cdot$ dist

$= 49000y(5-y) \, \Delta y$ J

$$W = \int_0^3 49000y(5-y) \, dy = 514500 \text{ J}$$

6. The hemispherical tank shown is filled with gasoline. Given that it has a radius of 4 ft and that gas weighs 42 lb/ft$^3$, find the work required to pump part of the gasoline out of the tank: Leave a depth of 1 ft of gasoline at the bottom of the tank.




$0 - y = -y$

$x^2 + y^2 = 4^2$

$x^2 = 16 - y^2$

dist $= -y$

& $-3 \le y \le 0$

$\Delta V = \pi r^2 \Delta y = \pi x^2 \Delta y$

$= \pi(16 - y^2) \, \Delta y$ ft$^3$

$\Delta F = (\text{unit weight}) \cdot \Delta V$

$= (42 \text{ lb/ft}^3)(\pi(16-y^2) \, \Delta y \text{ ft}^3)$

$= 42\pi(16-y^2) \, \Delta y$ lb

$\Delta W = \Delta F \cdot$ dist

$= 42\pi(16-y^2)(-y) \, \Delta y$ ft-lb

$$W = \int_{-3}^{0} 42\pi(16-y^2)(-y) \, dy = \frac{4347\pi}{2} \text{ J} = 6828.25 \text{ J}$$