Key

Homework Set 4

(sect 5.2 & 5.3: Evaluating Definite Integrals)

1. If $\int_1^4 \sqrt{x}\, dx = \frac{14}{3}$, what is $\int_4^1 \sqrt{x}\, dx$?

   $-14/3$

2. Evaluate: $\int_0^0 x^2 \cos x\, dx$

   0

3. Write as a single integral in the form $\int_a^b f(x)\, dx$:

$$\int_{-2}^{3} f(x)\, dx + \int_{3}^{7} f(x)\, dx - \int_{-2}^{-1} f(x)\, dx = \int_{-1}^{7} f(x)\, dx$$

4. If $\int_{-1}^4 f(x)\, dx = 13$ and $\int_{-1}^2 f(x)\, dx = 7.4$, find $\int_2^4 f(x)\, dx$.

   5.6

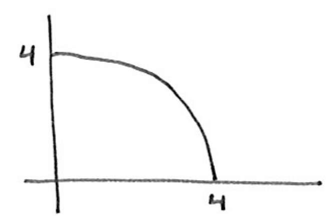
5. If $\int_2^5 f(x)\, dx = 4$ and $\int_2^5 g(x)\, dx = 5$, find $\int_2^5 [3f(x) - 2g(x)]\, dx$.

   $3(4) - 2(5) = 2$

Evaluate the definite integral by interpreting it in terms of area. Draw the graph of the function.

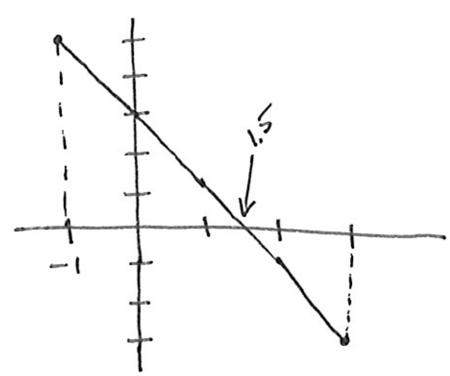
6. $\int_0^4 \sqrt{16 - x^2}\, dx$

$$= \frac{1}{4}\left[\pi \cdot 4^2\right]$$
$$= 4\pi$$



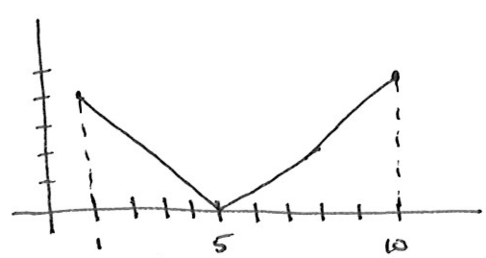
7. $\int_{-1}^3 (3 - 2x)\, dx$

$$= \frac{1}{2}(5)(2.5) + \frac{1}{2}(-3)(1.5)$$
$$= 4$$



8. $\int_1^{10} |x - 5|\, dx$

$$= \frac{1}{2}(4)^2 + \frac{1}{2}(5)^2$$
$$= 20.5$$



Evaluate the definite integrals using antiderivatives.

9. $\int_0^3 (1+2u-4u^3)\,du$

$$= u+u^2-u^4\Big|_0^3 = (3+9-81)-0 = -69$$

10. $\int_0^1 x(\sqrt[3]{x}-\sqrt[4]{x})\,dx$ $= \int_0^1 x^{4/3} - x^{5/4}\,dx$

$$= \frac{3}{7}x^{7/3} - \frac{4}{9}x^{9/4}\Big|_0^1 = \left(\frac{3}{7}-\frac{4}{9}\right) - 0 = -\frac{1}{63}$$

11. $\int_1^2 \frac{y+5y^7}{y^3}\,dy$ $= \int_1^2 y^{-2}+5y^4\,dy$

$$= -y^{-1}+y^5\Big|_1^2 = \left(-\frac{1}{2}+32\right)-(-1+1) = 31.5$$

12. $\int_0^4 (2t+5)(3t-1)\,dt$ $= \int_0^4 6t^2+13t-5\,dt$

$$= 2t^3+\frac{13}{2}t^2-5t\Big|_0^4 = (128+104-20)-0 = 212$$

13. $\int_0^5 (2e^x+4\cos x)\,dx$

$$= 2e^x+4\sin x\Big|_0^5 = (2e^5+4\sin 5)-(2e^0+0) = 2e^5+4\sin 5-2$$

OR $= 290.9906211$

14. $\int_1^9 \frac{dx}{2x}$ $= \frac{1}{2}\int_1^9 \frac{1}{x}\,dx$

$$= \frac{1}{2}\ln x\Big|_1^9 = \frac{1}{2}\ln 9 - \frac{1}{2}\ln 1 = \ln 3 \overset{\text{OR}}{=} 1.098612289$$

15. $\int_0^{\pi/4} (1+\sec^2 x)\,dx$

$$= x+\tan x\Big|_0^{\pi/4} = \left(\frac{\pi}{4}+\tan\left(\frac{\pi}{4}\right)\right)-0 = \frac{\pi}{4}+1 = \frac{4+\pi}{4} \overset{\text{or}}{=} 1.785398163$$

16. $\int_{-1}^1 e^{x+1}\,dx$

$$= e\int_{-1}^1 e^x\,dx = e\cdot e^x\Big|_{-1}^1 = e^{x+1}\Big|_{-1}^1$$

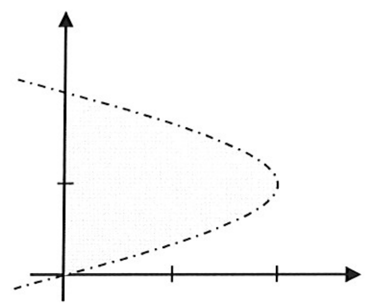
$$= e^2-e^0 = e^2-1 \overset{\text{OR}}{=} 6.389056099$$

17. What is wrong with this equation? $\int_{-1}^{3}\frac{1}{x^2}dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{3} = -\frac{4}{3}$

$$\int_{-1}^{3}\frac{1}{x^2}dx = \int_{-1}^{3}x^{-2}dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{3} = \left.-\frac{1}{x}\right|_{-1}^{3} = \left(-\frac{1}{3}\right)-\left(-\frac{1}{(-1)}\right) = -\frac{1}{3}-1 = -\frac{4}{3}$$

$f(x)=\frac{1}{x^2}$ is undefined at $x=0$

18. The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 4y - 2y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (4y-2y^2)\,dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 4y - 2y^2$ from $y = 0$ to $y = 2$.) Find the area of this region.



$$\int_0^2 (4y-2y^2)\,dy$$
$$= \left.2y^2-\frac{2}{3}y^3\right|_0^2$$
$$= (8-16/3)-0 = 8/3$$

19. The acceleration function $a(t) = 2t + 3$ in m/sec^2 and the initial velocity $v(0) = -4$ for a particle moving along a line are given. Find the velocity at time t and the distance traveled over the first 3 seconds.

$a(t) = 2t+3$

$v(t) = t^2+3t+C \quad , \quad v(0)=-4 \Rightarrow C=-4$

$v(t) = t^2+3t-4$

$$f(3)-f(0) = \int_0^3 v(t)\,dt = \int_0^3 t^2+3t-4\,dt$$
$$= \left.\frac{1}{3}t^3+\frac{3}{2}t^2-4t\right|_0^3 = \left(9+\frac{27}{2}-12\right)-0 = 10.5$$

20. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

$$\int_0^{10} r(t)\,dt = \int_0^{10} 200-4t\,dt$$
$$= \left.200t-2t^2\right|_0^{10}$$
$$= (2000-200)-0$$
$$= 1800$$