Key

Homework Set 5

(sect 5.4: The Fundamental Theorem of Calculus & the Average Value of a Function)

Use the Fundamental Theorem of Calculus to find the derivative.

1. $\frac{d}{dx}\int_0^x \sqrt{1+2t}\,dt \quad = \sqrt{1+2x}$

2. $\frac{d}{dx}\int_1^x \ln t\,dt \quad = \ln x$

3. $\frac{d}{dx}\int_x^5 \sec^2 t\,dt \quad = -\frac{d}{dx}\int_5^x \sec^2 t\,dt \quad = -\sec^2 x$

4. $\frac{d}{dx}\int_0^{x^2} \sqrt{1-t^3}\,dt \quad = 2x\sqrt{1-(x^2)^3} \quad = 2x\sqrt{1-x^6}$

5. $\frac{d}{dx}\int_{3x}^{5x} \frac{t^2-1}{t^2+1}\,dt$ $\qquad$ $\left[hint: \int_{ax}^{bx} f(t)\,dt = \int_0^{bx} f(t)\,dt + \int_{ax}^{0} f(t)\,dt\right]$

$$= \frac{d}{dx}\int_0^{5x} \frac{t^2-1}{t^2+1}\,dt - \frac{d}{dx}\int_0^{3x} \frac{t^2-1}{t^2+1}\,dt \quad = 5\cdot\frac{(5x)^2-1}{(5x)^2+1} - 3\cdot\frac{(3x)^2-1}{(3x)^2+1}$$

$$= 5\cdot\frac{25x^2-1}{25x^2+1} - 3\cdot\frac{9x^2-1}{9x^2+1}$$

Find the average value of the function on the given interval.

6. $f(x) = \frac{1}{x}$, $[1,4]$

$$f_{AVG} = \frac{1}{4-1}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}\ln x\Big|_1^4 = \frac{1}{3}\ln 4 = \frac{2}{3}\ln 2 \overset{\text{or}}{=} .4620981204$$

7. Let $f(x) = (x-3)^2$
   a. Find the average value of the function over $[2,5]$.

$$f_{AVG} = \frac{1}{5-2}\int_2^5 (x-3)^2\,dx = \frac{1}{3}\int_2^5 x^2-6x+9\,dx = \frac{1}{3}\left(\frac{1}{3}x^3-3x^2+9x\right)\Big|_2^5 = 1$$

   b. Find c such that $f_{AVG} = f(c)$.

$$1 = (x-3)^2$$
$$\pm 1 = x-3$$
$$3 \pm 1 = x$$

So $\quad C = 2$ or 4

   c. Sketch the graph of $f(x)$ and a rectangle whose area is the same as the area under the graph of f. (Draw both of these on the same coordinate plane and the interval $[2,5]$.)