# Homework Set 6

(sect 5.5: U-Substitution)

Evaluate the indefinite integrals.

1. $\int \cos 3x\, dx$

$= \frac{1}{3}\int \cos(3x)(3\,dx)$

$= \frac{1}{3}\int \cos u\, du = \frac{1}{3}\sin u = \frac{1}{3}\sin(3x) + C$

$u = 3x$

$du = 3\,dx$

2. $\int e^{\sin x}\cos x\, dx$

$= \int e^u du$

$= e^u = e^{\sin x} + C$

$u = \sin x$

$du = \cos x\, dx$

3. $\int xe^{x^2}\, dx$

$= \frac{1}{2}\int e^{x^2}(2x\,dx)$

$= \frac{1}{2}\int e^u\, du$

$= \frac{1}{2}e^u = \frac{1}{2}e^{x^2} + C$

$u = x^2$

$du = 2x\, dx$

4. $\int \frac{e^x}{e^x+1}dx$

$= \ln|e^x+1| + C$

$u = e^x + 1$

$du = e^x\, dx$

5. $\int \frac{1+x}{1+x^2}dx = \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}\,dx$

$= \arctan x + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$= \arctan x + \frac{1}{2}\ln(1+x^2) + C$

Evaluate the definite integrals.

6. $\int_0^2 (x-1)^{25}\, dx$

$= \int_{-1}^{1} u^{25}\, du$

$= \frac{1}{26}u^{26}\Big|_{-1}^{1}$

$= \frac{1}{26} - \frac{1}{26} = 0$

$u = x - 1$

$du = dx$

| $x$ | $u$ |
|---|---|
| 2 | $2-1=1$ |
| 0 | $0-1=-1$ |

7. $\int_0^1 x^2(1+2x^3)^5\,dx$

$= \frac{1}{6}\int_0^1 (1+2x^3)^5 (6x^2\,dx)$

$= \frac{1}{6}\int_1^3 u^5\,du$

$= \frac{1}{6}\cdot\frac{1}{6}u^6\Big|_1^3 = \frac{729}{36} - \frac{1}{36} = \frac{182}{9} = 20.\overline{2}$

$u = 1+2x^3$

$du = 6x^2\,dx$

| $x$ | $u$ |
|---|---|
| 1 | $1+2(1)^3 = 3$ |
| 0 | $1+2(0) = 1$ |

8. $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$

$= \int_1^4 \frac{1}{\sqrt{u}}\,du$

$= \int_1^4 u^{-1/2}\,du$

$= 2u^{1/2}\Big|_1^4 = 2\cdot 2 - 2\cdot 1 = 2$

$u = \ln x$

$du = \frac{1}{x}\,dx$

| $x$ | $u$ |
|---|---|
| $e^4$ | $\ln e^4 = 4$ |
| $e$ | $\ln e = 1$ |

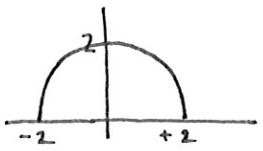
9. $\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx$ [hint: write as two integrals and evaluate one of them in terms of area]

$= \int_{-2}^{2} x\sqrt{4-x^2}\,dx + 3\int_{-2}^{2}\sqrt{4-x^2}\,dx$

$= -\frac{1}{2}\int_{-2}^{2}\sqrt{4-x^2}(-2x\,dx) + 3\left[\frac{1}{2}\cdot\pi\cdot 2^2\right]$

$= -\frac{1}{2}\int_0^0 \sqrt{u}\,du + 3\cdot 2\pi$

$= 0 + 6\pi = 6\pi$




$u = 4-x^2$

$du = -2x\,dx$

| $x$ | $u$ |
|---|---|
| -2 | $4-(-2)^2 = 0$ |
| 2 | $4-(2)^2 = 0$ |

10. A bacteria culture starts with an initial population of 400 bacteria and grows at a rate of $r(t) = (450.268)e^{1.12567t}$ bacteria per hour. How many bacteria will there be after 3 hours?

$r(t) = P'(t) = 450.268\,e^{1.12567t}$

$P(t) = \frac{450.288}{1.12567}e^{1.12567t} + C$, $P(0) = 400$

So, $P(t) = 400\,e^{1.12567t}$

$P(3) = 11{,}713.23347$

$\therefore$ 11713 bacteria after 3 hrs