Homework Set 7

(sect 6.1: Integration by Parts)

Evaluate the given integral.

1. $\int xe^{-x}\,dx$

$= -xe^{-x} - \int -e^{-x}\,dx$

$= -xe^{-x} - e^{-x} + C$

OR $-e^{-x}(x+1) + C$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = e^{-x} \qquad g(x) = -e^{-x}$

2. $\int x\cos(3x)\,dx$

$= \frac{1}{3}x\sin(3x) - \int \frac{1}{3}\sin(3x)\,dx$

$= \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos(3x) + C$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = \cos(3x) \qquad g(x) = \frac{1}{3}\sin(3x)$

3. $\int \ln(x+5)\,dx$

$= x\ln(x+5) - \int \frac{x}{x+5}\,dx$

$= x\ln(x+5) - \int \frac{x+5}{x+5} - \frac{5}{x+5}\,dx$

$= x\ln(x+5) - \int 1 - \frac{5}{x+5}\,dx = x\ln(x+5) - x + 5\ln(x+5) + C$

$f(x) = \ln(x+5) \qquad f'(x) = \frac{1}{x+5}$

$g'(x) = 1 \qquad g(x) = x$

4. $\int x^6\ln(x)\,dx$

$= \frac{1}{7}x^7\ln x - \int \frac{1}{7}x^6\,dx$

$= \frac{1}{7}x^7\ln x - \frac{1}{49}x^7 + C$

$f(x) = \ln x \qquad f'(x) = 1/x$

$g'(x) = x^6 \qquad g(x) = \frac{1}{7}x^7$

5. $\int \arctan\left(\frac{x}{4}\right)dx$

$= x\arctan(x/4) - \int \frac{4x}{x^2+16}\,dx$

$= x\arctan(x/4) - 2\ln(x^2+16) + C$

$f(x) = \arctan\left(\frac{x}{4}\right) \qquad f'(x) = \frac{4}{x^2+16}$

$g'(x) = 1 \qquad g(x) = x$

6. $\int_1^2 \frac{\ln(x)}{x^2}\,dx = \int_1^2 x^{-2}\ln x\,dx$

$= -\frac{1}{x}\ln x\Big|_1^2 + \int_1^2 x^{-2}\,dx$

$= -\frac{\ln x}{x} - x^{-1}\Big|_1^2$

$= \left(-\frac{\ln 2}{2} - \frac{1}{2}\right) - (0 - 1)$

$= \frac{1}{2} - \frac{\ln 2}{2}$

$f(x) = \ln x \qquad f'(x) = 1/x$

$g'(x) = x^{-2} \qquad g(x) = -x^{-1}$

7. $\int_0^\pi x^3 \sin(x/2)\,dx$

$$= -2x^3\cos(x/2) + 12x^2\sin(x/2) + 48x\cos(x/2) - 96\sin(x/2)\Big|_0^\pi$$

$$= (0 + 12\pi^2 + 0 - 96) - (0)$$

$$= 12\pi^2 - 96 \quad \text{OR} \quad 22.4352528$$

+/-	f	g'
+	x^3	$\sin(x/2)$
−	$3x^2$	$-2\cos(x/2)$
+	$6x$	$-4\sin(x/2)$
−	6	$8\cos(x/2)$
+	0	$16\sin(x/2)$

note: $\cos(\pi/2) = 0$
$\sin(\pi/2) = 1$

8. $\int x^5 e^x\,dx$

$$= x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120xe^x - 120e^x + C$$

$$\text{OR} = e^x(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120) + C$$

+/-	f	g'
+	x^5	e^x
−	$5x^4$	e^x
+	$20x^3$	e^x
−	$60x^2$	e^x
+	$120x$	e^x
−	120	e^x
+	0	e^x

9. $\int x^5 \sin(x^3)\,dx$

$u = x^3$
$du = 3x^2dx$

$$= \frac{1}{3}\int u\sin u\,du$$

$$= \frac{1}{3}\left[-u\cos u + \int \cos u\,du\right]$$

$$= -\frac{1}{3}x^3\cos x^3 + \frac{1}{3}\sin x^3 + C$$

$f(u) = u \qquad f'(u) = 1$
$g'(u) = \sin u \qquad g(u) = -\cos u$

10. $\int e^{2x}\cos(x)\,dx$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - \int 2e^{2x}\sin x\,dx$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - \left[-2e^{2x}\cos x + \int 4e^{2x}\cos x\right]$$

$f(x) = e^{2x} \qquad f'(x) = 2e^{2x}$
$g'(x) = \cos x \qquad g(x) = \sin x$

$f(x) = 2e^{2x} \qquad f'(x) = 4e^{2x}$
$g'(x) = \sin x \qquad g(x) = -\cos x$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x$$

$$5\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x$$

$$\int e^{2x}\cos x\,dx = \frac{1}{5}e^{2x}(\sin x + 2\cos x) + C$$